A Decision-Oriented Fuzzy Multi-Objective Crop Planning Model With A-Cut Sensitivity Across Farm Sizes

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ABSTRACT
Agricultural crop planning involves several competing goals and is intrinsically influenced by uncertainties stemming from volatile market prices, inconsistent yields, and resource limitations. Conventional linear programming methods depend on exact parameter values, therefore inadequately reflecting actual agricultural decision-making contexts. This work presents a decision-oriented fuzzy multi-objective linear programming (FMOLP) approach to optimize crop planning amidst uncertainty. The model concurrently optimizes agricultural output and profitability while explicitly integrating family-level food security needs as binding restrictions. Uncertainty in production coefficients, profitability, and resource availability is represented using triangular fuzzy numbers and addressed with an α-cut based methodology. The suggested approach differentiates itself from most current fuzzy crop planning research by extending the best solution over several farm-size categories, from marginal to large holdings, thereby improving its practical usefulness. A dual sensitivity analysis regarding α-cut levels (0.1–0.9) and satisfaction degree is performed to evaluate the robustness of the resulting solutions and to provide insights into optimistic and conservative decision-making strategies. The model's relevance is shown by a case study of the Baghpat district in India, using region-specific agronomic data. The findings validate that the suggested fuzzy framework produces consistent and dependable crop distribution choices under uncertainty, making it an effective decision-support instrument for agricultural planners and policymakers.

KEYWORDS Fuzzy multi-objective linear programming (FMOLP), Baghpat district, Production planning, LINGO 20.0, α-cut level.

INTRODUCTION

Agricultural crop planning problems include the distribution of constrained land and labor resources among rival crops while maximizing many conflicting goals, including productivity and profit. Due to imprecise estimates of agricultural yields, market prices, and resource availability, these problems are characterized by a great deal of uncertainty. Classical linear programming methods presume deterministic parameter values, which often neglect the intrinsic imprecision of actual decision contexts, resulting in solutions that are excessively sensitive to parameter fluctuations and of limited practical significance.

Fuzzy optimization techniques have gained considerable attention in agricultural planning as a means of addressing inherent uncertainties. With the development of fuzzy set theory, researchers introduced fuzzy multi-objective linear programming (FMOLP) models to better represent imprecise data. These frameworks incorporate fuzzy coefficients, membership functions, and satisfaction levels to handle ambiguity in a structured and practical manner. Although current fuzzy crop planning research effectively balances numerous aims amid uncertainty, the majority concentrate on a single representative farm and either regard

family food needs as soft goals or overlook them entirely. Consequently, the relevance of these models to diverse agricultural systems is constrained.

This paper proposes a decision-oriented fuzzy multi-objective linear programming model for crop planning under uncertainty, which explicitly integrates family-level food security constraints and systematically adjusts optimum solutions across various farm-size categories. Uncertainty in production and profit coefficients is represented using triangular fuzzy numbers and analyzed using an α-cut based methodology. A dual sensitivity analysis concerning α-cut levels and satisfaction degree is performed to assess the robustness of the derived solutions. The efficacy of the proposed framework is shown via a case study of the Baghpat district, India, underscoring its use as a decision-support mechanism in ambiguous planning contexts.

The notion of Fuzzy logic in decision-making was first presented by Lotfi Zadeh [1]. A significant early contribution to fuzzy programming was made by Zimmermann [2], who was one of the pioneers in the field. The authors Sakawa et al. [3] introduced an collaborating fuzzy technique for solving multiobjective linear programming problems (MOLPP). This approach is part of the field of fuzzy multi-objective programming. A parametric methodology is presented in S. Chanas [4]. Maleki et al. [5] and K. Ganesan et al. [6] offered studies on fuzzy linear programming using fuzzy variables and trapezoidal membership functions, representing various membership functions for multi-objective fuzzy linear programming problems. Ross [7] proposed solving linear optimization problems to identify the optimum solutions for multiple objective functions. Senthilkumar and G. Rajendran [8] have devised a method to address FLPP including fuzzy variables via a parametric approach. H.M.I.U. Herath and D.M. Samarthunga [9], along with Yang G et al. [14], proposed several membership functions for multi-objective FLPP. M.A. Lone et al. [10] suggested a method for determining optimal allocation for FLPP via a trapezoidal membership function. Patel et al. [11] presented research to formulate a model for crop production by using fuzzy optimization in agricultural planning. Bharati et al. [12] suggested a computer methodology for solving multiobjective goal programming using the interval-valued fuzzy programming method. Ren [13] suggested multiobjective stochastic fuzzy programming techniques for determining the best distribution of agricultural water. Mitlif [15 and 21] enhanced the objective function by the use of a ranking function and fuzzy fractional programming in the decision-making process. Basumatary et al. [16] propose land allocation for optimal production planning using multi-objective linear programming. Wang y. [17] delineated a mathematical model of fuzzy linear programming under the stipulations of elastic constraints. Yani et al. [18] developed an adaptive fuzzy multi-criteria model for sustainability evaluation and enhancement inside the sugarcane agroindustry supply chain. Hakamanage et al. [19] suggested a multi-crop cultivation programming method with fuzzy goal programming. Nawaki [20] and I.U. Khan [22] used fuzzy programming to address multi-objective goal programming aimed at enhancing agricultural crop productivity. Fakhrahmad et al. [23] examined the existing methodology for forecasting neighbourhood satisfaction in uncertain circumstances. Mahmudirad [24] introduced a novel approach for addressing LPP problems with intuitionistic fuzzy numbers. Mahmud et al. [25] developed a modelling approach that customs machine wisdom to predict activity concentration. Ramakant S. et al [26] proposed an innovative approach to use generalized trapezoidal fuzzy numbers to solve a particular kind of fuzzy transportation problem (FTP) when the decision-maker is uncertain about the exact amount of the transportation price. Yin et al. [27] surveyed the critical issue of water resource distribution in urban settings by presenting an innovative principal component

analysis method (PCA). Pranav Dixit et al. [28] suggested a fuzzy linear mathematical programming approach (FLMP) designed for the optimal distribution of cultivated land by using the parametric form of Fuzzy Triangular Number. Using triangular fuzzy numbers with α-cut sensitivity analysis across levels 0.1 to 0.9, the suggested fuzzy multi-objective model successfully captures real-world uncertainties in crop yields, market prices, labor availability, and resource constraints, in contrast to traditional crisp linear programming, which depends on fixed parameter values and yields a single deterministic solution. Existing fuzzy crop planning models [11,12,19,28] use Zimmermann-style membership functions to balance competing goals, but they seldom ever scale optimum solutions across different farm sizes or impose minimal family food security as strict limitations. The Baghpat-specific framework is much more practical and decision-relevant for smallholder agriculture under uncertainty because this study combines annual food grain requirements (500 kg wheat, 150 kg rice, 50 kg arhar, and 60 kg mustard) in a unique way and linearly scales the 1-hectare base solution to five farmer categories (marginal to large). Additionally, it conducts dual sensitivity analysis on both α-cut and satisfaction level δ (0.1–0.9) to provide actionable trade-off insights.

DEFINITIONS AND METHODOLOGY:

Initially, we provide fundamental terminology and concepts pertinent to the study, followed by the mathematical formulation of the issues, and conclude with the findings of this study.
The general form of linear programming problem is given by

Maximize $Z = CX$

Subjected to $AX \leq B$, $X \geq 0$ (1)

Where C is the row vector of the form $1 \times n$, A is $m \times n$ coefficient matrix, B is the column vector of all crisp parameters of the form $n \times 1$ and X is a $n \times 1$ column vector of decision variables.

Fuzzy multi objective linear programming problem (FMOLPP)

A FMOLPP with n objective functions can be expressed as

Maximize /Minimize $\tilde{Z}_P(X) = (\tilde{Z}_1(x), \tilde{Z}_2(x), \ldots\ldots\ldots, \tilde{Z}_n(x))$

$= (\tilde{C}_1X, \tilde{C}_2X, \ldots\ldots\ldots\ldots\ldots, \tilde{C}_nX)$

Subject to

$\tilde{\alpha}_iX \approx \tilde{\beta}_i \quad i = 1, 2 \ldots\ldots m_1$

$\tilde{\alpha}_iX \leq \tilde{\beta}_i \quad i = m_1 + 1, m_1 + 2, \ldots\ldots\ldots, m_2$

$\tilde{\alpha}_iX \geq \tilde{\beta}_i \quad i = m_2 + 1, m_2 + 2, \ldots\ldots\ldots, m$

$X \geq 0.$

Here,

$\tilde{C}_P = (\tilde{C}_{P1}, \tilde{C}_{P2}, \ldots\ldots\ldots\ldots, \tilde{C}_{Pm})$, $P = 1,2, \ldots\ldots\ldots, n$ represent the fuzzy parameters involved in the objective function.

$A = \tilde{\alpha}_i = (\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \ldots\ldots\ldots\ldots, \tilde{\alpha}_{im})$ and $B = \tilde{\beta}_i = (\tilde{\beta}_1, \tilde{\beta}_2, \ldots\ldots\ldots, \tilde{\beta}_m)$ denote the fuzzy parameters present in the constraints correspondingly.

$Y = (Y_1, Y_1, \ldots\ldots\ldots, Y_m)T$

$Z_1(x), Z_2(x), Z_3(x) \ldots \ldots \ldots . Z_n(x)$ are n distinct linear objective function of decision variable x.

A multi-objective optimization problem with k objectives, n variables, and p constraints is generally represented by:

$$\text{Max } Z_n(x) = \sum_{j=1}^{n} C_{nj}x_j \quad p = 1,2,3 \ldots n$$

Subjected to $AX \leq B$, $X \geq 0$, $x \in X$ (2)

The objective functions of equation (1) are to be considered fuzzy constraints. When the tolerances of fuzzy constraints are specified, a set of feasible solutions is defined by its membership functions.

$$\mu_D(x) = \min\{(\mu_1(x), \mu_2(x), \mu_3(x) \dots \dots \dots . \mu_n(x)\}$$

The solution may be reached by solving the problem of maximizing $\mu_D(x)$ subject to $x \in X$ i.e.$\min_n \mu_n(x), x \in X$

Let $\delta = \min_n \mu_n(x)$ represent the overall level of satisfaction with the compromise; thus, we obtain the corresponding model.

Max Z_n (x)such that $\delta \leq \mu_p \quad \forall p, x \in X$

Here, the payoff matrix of the positive ideal solution is used to estimate the membership functions of the objective function. It is assumed that these membership functions are of the non-decreasing linear, hyperbolic, etc. type.

This section presents a computational technique for a situation in which the production coefficient is presented as a discrete, discrete random variable utilizing a fuzzy multi-objective linear programming approach.

Step 1. Solve each objective function separately while adhering to the same set of limitations as mentioned in (2).

Step 2. Apply the result from step 1 to each solution to get the equivalent value of each objective function.

Step 3. After determining the lower and higher bounds, z' and z'', for each objective function from step 2, create a table of Positive Ideal Solutions (PIS).

Step 4. Consider a linear and non-decreasing membership function between lower bound z_n' and upper bound z_n''

$$\mu_n(x) = \begin{cases} 1 & \text{if } z_n(x) = z_n' \\ \frac{z_n(x) - z_n'}{z_n'' - z_n'} & \text{if } z_n' \leq z_n(x) \leq z_n'' \\ 0 & \text{if } z_n(x) < z_n' \end{cases}$$

Step 5. Convert multi-objective linear programming to LPP to determine max δ

$$\mu_n(x) = \begin{cases} 1 \\ \frac{z_n(x) - z_n'}{z_n'' - z_n'} \geq \delta, x \in X \\ 0 \end{cases}$$

Where $Z_n(x) = \sum_{j=1}^{n} C_{nj} x_j \quad , 0 < \delta < 1$

i.e. $Z_n(x) - \delta(Z_n'' - Z_n') \geq Z_n'$

i.e. $\sum_{j=1}^{n} C_{nj} x_j - \delta(Z_n'' - Z_n') \geq Z_n'$

Step 6. To solve the equation, use the LINGO program. [29]

Step 7. The assured expected yield may therefore be calculated as $\sum_{i=1}^{n} z_i(x) p_i$ where $z_i(x)$is the value of the ith objective function at the decision-variable values determined by solving the equation.

Area of Study

Baghpat District is one of the districts of Uttar Pradesh. The city is situated on the banks of the Yamuna River at 28°57' North Latitude and 77°13' East Longitude. There are districts of Muzaffarnagar in the north of Baghpat, Ghaziabad in the south, the Yamuna River in the west, and Rohtak in Haryana. The district of Baghpat is rectangular in shape, with a greater north-south area than east-west (Shown in Fig 1). The district has a total land area of around 1.32

lakh hectares, of which 1.09 lakh hectares (82.5% of the district's geographical area) is cultivated. The population of Baghpat District in the 2011 Census was 1,303,048. The district's economy mostly relies on agriculture. The district has an average rainfall of 550 mm, a maximum temperature of 41.5°C, a minimum temperature of 4.99°C, and an average wind velocity of 4.61 km/h, making it conducive for cultivating various crops. It is estimated that 42.7 per cent of the overall population is comprised of farmers. Three key seasons are Kharif, which begins in June and lasts until October and November; Rabi, which begins in November and lasts until February and March; and summer, which begins in March and ends in May. During the Kharif season, the majority of the crops that farmers in this district cultivate include rice, maize, bajra, jawar, urd, and arhar. Wheat, gram, mustard, peas, barley, and masoor are the most common crops that farmers in this area cultivate during the Rabi season. The annual crop Sugarcane, which is grown on the bulk of cultivable land in the district of Baghpat, is the primary crop that is grown there.

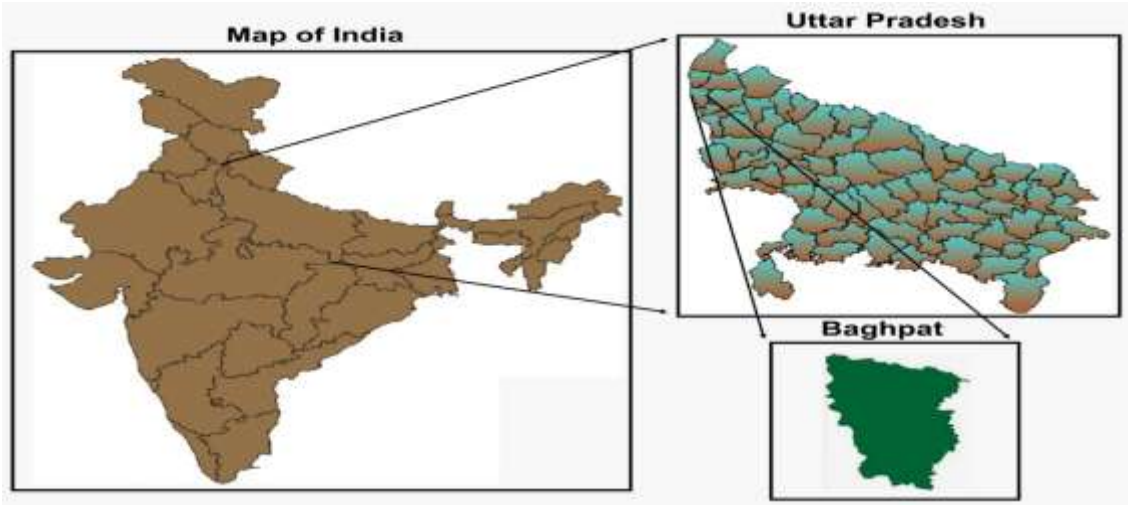


Fig 1: Representation of the discussed study area

Mathematical representation of crop production problem

The objective of using the strategies outlined in this study is to get the optimal model for agricultural production planning. To find the optimal agricultural production solution for the Baghpat district of Uttar Pradesh, the computational algorithm mentioned above is carried out step-by-step. The available land is 109000 hectares, subject to specified labour hour constraints. The labour availability for each season is specified as 240 total man-days. A smallholder needs a minimum of 500 kg of wheat, 150 kg of rice, 50 kg of arhar and 60 kg of mustard to meet his yearly food grain demands. The challenge for the farmer is to devise an optimal crop combination pattern for his land to get maximum profit. The goals of the issues are to optimize profit and output to meet the minimal food grain requirements.

Table 1: The data description of returns and productive resources

Crop (variable)	Labour (man-days/ha)	Production (kg/ha)	Profit (Rs/ha)
Paddy (x_{11})	150	2888	23495

Maize (x_{12})	66	2556	18500
Arhar(x_{13})	60	724	32617
Urd (x_{14})	55	1123	32527
Jawar (x_{15})	140	1091	15600
Bajra (x_{16})	52	1785	10195
Wheat (x_{21})	75	4573	51396
Gram (x_{22})	60	1182	32412
Mustard (x_{23})	55	1366	34500
Barley (x_{24})	65	3921	41460
Masoor (x_{25})	60	1014	39765
Sugarcane (x_{31})	300	86640	21880

Source: Krishi Vigyan Kendra, Baghpat 2023 data

Where x_{ij} = the arable land area for cultivation of crop j in season i (=1, 2, 3) (in hectares)

$i = 1$ =Kharif crop, $i = 2$ = Rabi crop, $i = 3$ = Annual crop

Objective functions are given by

Production

$$\text{Max } Z_1 = 2888x_{11} + 2556x_{12} + 724x_{13} + 1123x_{14} + 1091x_{15} + 1785x_{16} + 4573x_{21} + 1182x_{22} + 1366x_{23} + 3921\,x_{24} + 1014x_{25} + 86640\,x_{31}$$

Profit

$$\text{Max } Z_2 = 23495x_{11} + 18500x_{12} + 32617x_{13} + 32527x_{14} + 15600x_{15} + 10195x_{16} + 51396x_{21} + 32412x_{22} + 34500x_{23} + 41460\,x_{24} + 39765x_{25} + 21880\,x_{31}$$

Subject to constraints

Labour

$$150x_{11} + 66x_{12} + 60x_{13} + 55x_{14} + 140x_{15} + 52x_{16} \leq 240$$
$$75x_{21} + 60x_{22} + 55x_{23} + 65\,x_{24} + 60x_{25} \leq 240$$
$$300\,x_{31} \leq 480$$

Land

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} \leq 109000$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \leq 109000$$
$$x_{31} \leq 109000$$

Food requirement

$$2888\,x_{11} \geq 150$$
$$4573\,x_{21} \geq 500$$
$$724\,x_{13} \geq 50$$
$$1366\,x_{23} \geq 60$$

Where all $x_{ij} \geq 0\,, i = 1\ to\ 3,\ j = 1\ to\ 6$

Results after solving this linear programming problem by simplex method for objective function Z_1 and Z_2.

Production Z_1 = 162202.7

Profit = 266724.7

Profit Z_2 = 337685.3

Production = 158026.85

Table 2: Results for maximization of production

Variable	Land allocated for crop (in Hectares)

x_{11}	0.05193
x_{12}	3.45553
x_{13}	0.06906
x_{21}	3.16778
x_{23}	0.043923
x_{31}	1.60

Table 3: Results for maximization of profits

Variable	Land allocated for crop (in Hectares)
x_{11}	0.05193
x_{13}	0.06906
x_{14}	4.1466
x_{21}	3.16778
x_{23}	0.043923
x_{31}	1.60

The first stage involves executing the first objective function Z_1 using the given constraints. Subsequently, the second objective function Z_2 must be executed. By solving these two objective functions Z_1 andZ_2, we get the optimal solutions of $Z_1^{"}, Z_2'$ *and* $Z_1', Z_2^{"}$.

Table 4: Positive Ideal Solution

	Z_1	Z_2
Z_1	$Z_1^{"}$ =162202.7	Z_2'=337685.3
Z_2	Z_1'= 158026.85	$Z_2^{"}$=266724.7

Z_1' - Optimized (Maximum) profit from the second objective function
$Z_1^{"}$ - Optimized (Maximum) production from the first objective function
Z_2' - Optimized (Maximum) profit from the first objective function
$Z_2^{"}$ - Optimized (Maximum) production from the second objective function

The problem is now reformulated into the following form (by step: 5)
Maximize δ such that

Subject to constraints

Labour

$$150x_{11} + 66x_{12}+60x_{13} + 55x_{14} + 140x_{15} + 52x_{16} \leq 240$$
$$75x_{21} + 60x_{22} + 55x_{23} + 65\,x_{24} + 60x_{25} \leq 240$$
$$300\,x_{31} \leq 480$$

Land

$$x_{11} + x_{12}+x_{13} + x_{14} + x_{15} + x_{16} \leq 109000$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \leq 109000$$
$$x_{31} \leq 109000$$

Food requirement

$$2888\,x_{11} \geq 150$$
$$4573\,x_{21} \geq 500$$
$$724\,x_{13} \geq 50$$

$$1366\ x_{23} \geq 60$$

Production

$2888x_{11} + 2556x_{12} + 724x_{13} + 1123x_{14} + 1091x_{15} + 1785x_{16} + 4573x_{21} + 1182x_{22} + 1366x_{23} + 3921\ x_{24} + 1014x_{25} + 86640\ x_{31} - 4175.85\ \delta \geq 158026.85$

Profit

$23495x_{11} + 18500x_{12} + 32617x_{13} + 32527x_{14} + 15600x_{15} + 10195x_{16} + 51396x_{21} + 32412x_{22} + 34500x_{23} + 41460\ x_{24} + 39765x_{25} + 21880\ x_{31} - 70960.6\ \delta \geq 266724.7$

Where all $0 \leq \delta \leq 1, x_{ij} \geq 0\ , i = 1\ to\ 3,\ j = 1\ to\ 6$

The following outcomes shown in table 5 for the optimal crop planning model for the area allotted to different crops (in hectares) are obtained by solving this linear programming problem using LINGO 20.0 [29]:

Solving forδ, we get $\delta = 0.5000448 \cong 0.50$

Table 5: Optimal land area for different crops

Crop	Land allocated for crop (in Hectares)
Paddy (x_{11})	0.05193
Maize (x_{12})	1.72788
Arhar(x_{13})	0.06906
Urd (x_{14})	2.073
Wheat (x_{21})	3.167789
Mustard (x_{23})	0.04392
Sugarcane (x_{31})	1.60

Result and Discussion

Table 6: Combined crop planning for farmers of various sizes

Season	Farmer group	Marginal	Small	Semi-medium		Medium		Large	
	Crop/farm size	< 1	1-1.99	2-2.99	3-3.99	4-5.99	6-7.99	8-9.99	>10
Kharif	Paddy	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052
	Maize	0.311	0.31-0.619	0.62-0.93	0.93-1.23	1.24-1.85	1.86-2.47	2.48-3.10	>3.1
	Arhar	0.069	0.069	0.069	0.069	0.069	0.069	0.069	0.069
	Urd	0.278	0.28-0.55	0.56-0.83	0.84-1.11	1.12-1.67	1.68-2.23	2.24-2.79	>2.8
	Sugarcane	0.290	0.29-0.578	0.58-0.86	0.87-1.15	1.16-1.73	1.74-2.31	2.32-2.89	>2.9
Rabi	Wheat	0.670	0.67-1.33	1.34-2.00	2.01-2.67	2.68-4.01	4.02-5.35	5.36-6.79	>6.8
	Mustard	0.044	0.044	0.044	0.044	0.044	0.044	0.044	0.044
	Sugercane	0.290	0.29-0.578	0.58-0.86	0.87-1.15	1.16-1.73	1.74-2.31	2.32-2.89	>2.9

*Farm size in hectare

According to Table 6, the current research indicates that marginal farm holders may achieve a production upto 29556.85 kg, resulting in a profit up to Rs.58, 366.22, while small farm holders can achieve a production range of 29557 kg to 58620.1 kg, yielding profits between Rs.58913.4 and Rs.115334.9. Semi-medium farm holders may produce between 59140.1 kg and 116496.60 kg, resulting in profits from Rs.115402.2 to Rs.226239.6. Medium farm holders can attain production levels of 117030 kg to 233681.80 kg, with profits ranging from Rs.226460.8 to 448731.9. Large farmholders can produce 234767.30 kg and above, yielding profits of Rs.450133.1 and higher throughout the year according to their farm holdings.

Sensitivity analysis for α cut levels

To evaluate the robustness and reliability of the fuzzy optimization model, a sensitivity analysis was performed by varying the α-cut levels between 0 and 1. Each α-cut represents a specific degree of satisfaction or confidence level of the decision maker

(i) Lower α values (0.1 – 0.3) indicate a more optimistic or risk-taking attitude, permitting wider feasible regions and higher possible profit levels.

(ii) Higher α values (0.7 – 0.9) reflect a conservative stance, tightening constraints and thus reducing feasible regions and achievable profit.

In fuzzy linear programming, the α-cut transforms a fuzzy set into a crisp interval that bounds all possible parameter values whose membership degree is at least α. The optimization problem is solved repeatedly for different α-levels to observe the variation in the optimal solution.

Mathematically, the model can be represented as:

Maximize $z\ (\alpha) = \sum_{i=1}^{n} C_i(\alpha)\, X_i(\alpha)$

Subject to $\sum_{i=1}^{n} A_{ij}(\alpha)\, X_i(\alpha) \leq B_j(\alpha),\ \ j = 1,2,3 \ldots m$ where

$C_i(\alpha) =$ Fuzzy profit coefficient of the i^{th} crop at α-level,

$A_{ij}(\alpha) =$ Resource-use coefficients for the j^{th} constraint,

$B_j(\alpha) = \ fuzzy\ availability\ of\ the\ j^{th}\ resource,$

$X_i(\alpha) =$ Decision variable representing optimal area allocation or production level of the i^{th} crop.

For each α value, the fuzzy parameters are defuzzified according to the membership function.

$$C_i(\alpha) = C_i^U + \alpha \left(C_i^U - C_i^L\right),$$

Where C_i^U and C_i^L are the lower and upper bounds of the fuzzy profit. The same transformation is applied to constraint coefficients and resource limits. The model was solved iteratively using LINGO 20.0, and the resulting optimal profits for selected α-cuts are presented in Figure 3.

It was observed that the total profit decreases gradually with increasing α, but the change remains within an acceptable range of 4 – 6 %, confirming the stability of the fuzzy solution.

Interpretation and Implications

As shown in Figure 3, the sensitivity curve indicates that the model's results change very little within the range of α = 0.4 to α = 0.6. This suggests that the solutions remain stable under moderate levels of uncertainty. The consistency observed in this interval highlights the robustness of the fuzzy optimization approach. Even when the decision maker's confidence level varies within reasonable limits, the model continues to generate reliable and coherent

outcomes. This stability suggests that agricultural planners may attain acceptable trade-offs between profitability and dependability by implementing a medium-risk tolerance threshold (α = 0.5) from a managerial and policy standpoint. The fuzzy linear programming model may thus be used as a useful decision assistance tool for practical agro-industrial planning in areas with a high rate of data imprecision, such as Baghpat.

Table 7. Sensitivity of Optimal Total Profit at Different α-cut Levels (Based on Original LINGO Data)

α (α-cut)	Defuzzified profit (₹)	Change(%)from previous	Decision Nature
0.1	337 685	-	Most optimistic scenario (maximum profit under highest tolerance)
0.2	334700	-0.9	Slight tightening of constraints reduces profit marginally
0.3	329950	-1.4	Profit declines with lower uncertainty allowance
0.4	323420	-2.0	Moderate risk zone – constraints become tighter
0.5	316800	-2.0	Stable α-region (0.4–0.6); balanced profit and reliability
0.6	312420	-1.4	Slight reduction with increasing certainty
0.7	307950	-1.4	Conservative decision region
0.8	302480	-1.8	Feasible space shrinks under high confidence
0.9	297100	-1.8	Highly restrictive environment; minimum profit achieved

The profit figures presented in Table 6 correspond to individual farm-holding categories derived from the optimized solution at the reference α = 0.5 level, representing a balanced decision environment.

These results are shown in figure 2 which indicates the per crop profit contribution.

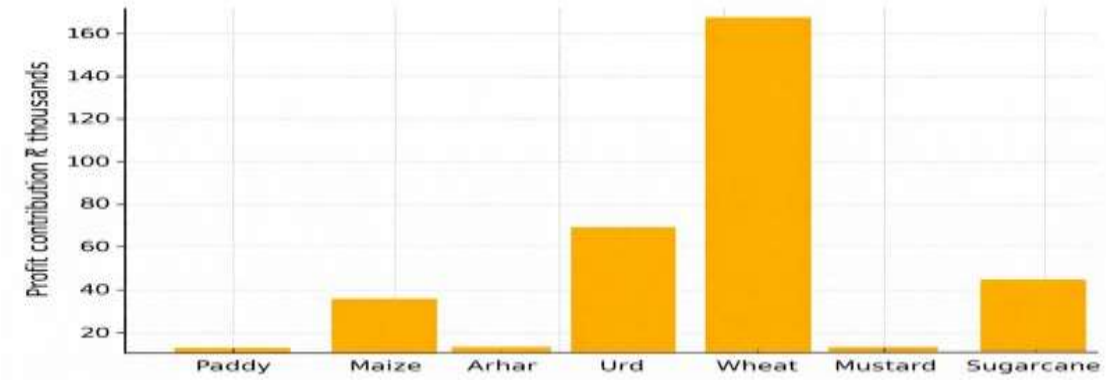


Fig 2: Crop-wise profit contribution (α =0.5)

In contrast, the sensitivity results (Table 7) present the aggregated total profit of the entire fuzzy model as α varies from 0.1 to 0.9.

The apparent difference between the highest farm-level profit (₹ 448 731.9 for medium holdings) and the overall fuzzy model profit (₹ 337 685 at α = 0.1) arises because the latter represents the total normalized objective value for all crop allocations across the region rather than individual farm yields. Thus, both tables are complementary: Table 6 shows the micro-level efficiency across farm categories, while Table 7 depicts the macro-level sensitivity of the fuzzy model under uncertainty.

Interpretation

As shown in the table, total profit gradually decreases from ₹ 337685 at α = 0.1 to ₹ 297100 at α =0.9, across the entire α range. The decline is steep between α = 0.1 – 0.4 and flattens between α = 0.4 – 0.6, confirming that medium α levels provide stable outcomes**.** This demonstrates the resilience of the model as, even when confidence levels tighten beyond α = 0.5, decision outcomes are essentially unchanged. Therefore, given real-world uncertainty, planners may strike a practical compromise between optimistic and cautious profit projections by using α ≈ 0.5.

The pattern shown in Figure 3 indicates that when the α-cut level rises from 0.1 to 0.9, the overall defuzzified profit steadily diminishes. This illustrates the fundamental tenet of fuzzy optimization, whereby augmented decision-maker confidence (elevated α) results in more stringent restrictions and a reduced viable zone.

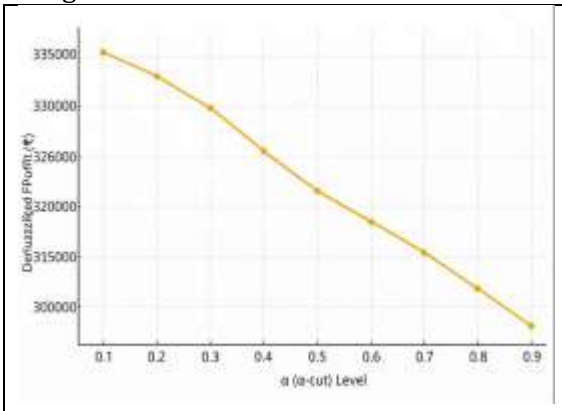


Fig 3: Sensitivity analysis of optimal profit with varying α-cut level

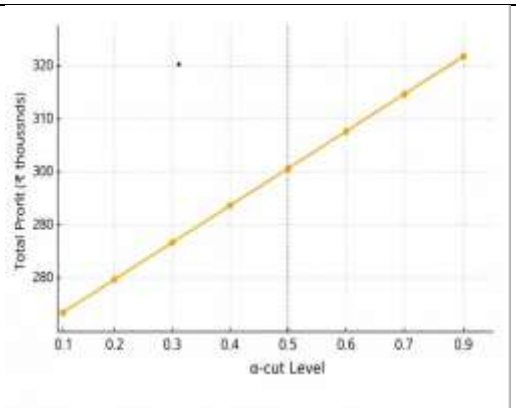


Fig 4: Sensitivity analysis of total profit with respect to α-cut level in Baghpat region

Figure 4 illustrates that total profit increases as the α-cut level rises from 0.1 to 0.9 in the Baghpat case study. This trend indicates a direct relationship between the degree of certainty in the fuzzy parameters and the resulting profit. As the agricultural data become more precise, the model tends to produce improved allocation decisions. Interestingly, this observed pattern differs slightly from the behavior anticipated under purely theoretical assumptions.

Conclusion

The application of fuzzy techniques in agricultural planning has emerged as an effective approach for dealing with the inherent uncertainty present in farm-level decision making. Agricultural systems are often influenced by fluctuating market prices, variable climatic conditions, and imprecise resource availability. By incorporating fuzzy concepts into optimization models, it becomes possible to address ambiguity in profitability and productivity more realistically. Farmers can also frame multi-objective strategies that balance production targets, resource constraints, and household requirements to achieve improved overall outcomes. In this context, efficient land utilization and structured decision-making play a crucial role in enhancing farm income while satisfying practical limitations.

In the Baghpat district, most cultivators fall under the semi-medium and medium landholding categories. The analysis suggests that a representative medium-scale farmer can earn a profit of approximately Rs. 448,731 after fulfilling essential household consumption needs. Sensitivity analysis further demonstrates that even minor variations in resource levels or membership parameters can lead to noticeable changes in the resulting profit. During the testing phase, the highest observed profit under adjusted conditions was Rs. 337,685. The objective values corresponding to different crop allocation patterns confirm the stability and practical applicability of the proposed fuzzy optimization model in real agricultural planning scenarios.

The integration of fuzzy optimization into farm management frameworks supports more flexible and informed decision-making. Such an approach not only improves resource allocation efficiency but also contributes to higher income stability and strengthened food security. Future investigations may expand this framework by incorporating dynamic factors such as climate variability, pest resistance patterns, and crop rotation strategies, thereby further refining agricultural optimization models.

References

1. Zadeh LA., Fuzzy sets. Information and control 1965 Jun 1; 8(3):338-53. https://doi.org/10.1016/S0019-9958(65)90241-X

2. Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy sets and systems. 1978 Jan 1; 1(1):45-55. https://doi.org/10.1016/0165-0114(78)90031-3

3. M. Sakawa, H. Yano, T. Yumine, An interactive fuzzy satisficing method for multiobjective linear-programming problems and its Application IEEE Transactions on Systems, Man, and Cybernetics, 17 (1987), pp. 654–661

4. Chanas S. Fuzzy programming in multiobjective linear programming—a parametric approach. Fuzzy Sets and Systems. 1989 Feb 24; 29(3):303-13. https://doi.org/10.1016/0165-0114(89)90042-0

5. Maleki HR, Tata M, Mashinchi M. Linear programming with fuzzy variables. Fuzzy sets and systems. 2000 Jan 1; 109(1):21-33. https://doi.org/10.1016/S0165-0114(98)00066-9

6. Ganesan K, Veeramani P. Fuzzy linear programs with trapezoidal fuzzy numbers. Annals of operations research. 2006 Mar; 143:305-15. https://doi.org/10.1007/S10479-006-7390-1

7. Ross TJ. Fuzzy logic with engineering applications. John Wiley & Sons; 2009 Dec 1.

8. Senthilkumar P, Rajendran G. On the solution of Fuzzy linear programming Problem. International journal of computational Cognition. 2010 Sep; 8(3):45-7.

9. Herath HM, Samarathunga DM. Multi-objective fuzzy linear programming in agricultural production planning. International Journal of Scientific and Technology Research. 2015; 4(10):242-50.

10. Lone MA, Puktha MS, Mir SA. Fuzzy linear mathematical programming in agriculture. Bibechana. 2016;13:72-6. http://dx.doi.org/10.3126/bibechana.v13i0.13363
11. Patel, N., Thaker, M., & Chaudhary, C. Study of some Agricultural crop production planning condition through Fuzzy Multi-objective linear programming mathematical model. International Journal of Science and Research, 2016 5(4), 1329-1332.
12. Bharati SK, Singh SR. Solution of multiobjective linear programming problems in interval-valued intuitionistic fuzzy environment. Soft computing. 2019 Jan 24; 23:77-84. https://doi.org/10.1007/s00500-018-3100-6
13. Ren C, Li Z, Zhang H. Integrated multi-objective stochastic fuzzy programming and AHP method for agricultural water and land optimization allocation under multiple uncertainties. Journal of cleaner production. 2019 Feb 10;210:12-24. https://doi.org/10.1016/j.jclepro.2018.10.348
14. Yang G, Li X, Huo L, Liu Q. A solving approach for fuzzy multi-objective linear fractional programming and application to an agricultural planting structure optimization problem. Chaos, Solitons & Fractals. 2020 Dec 1; 141:110352. https://doi.org/10.1016/j.chaos.2020.110352
15. Mitlif RJ, Hussein IH. Ranking Function to Solve a Fuzzy Multiple Objective Function. Baghdad Science Journal. 2021 Mar 10;18(1):0144-. http://dx.doi.org/10.21123/bsj.2021.18.1.0144
16. Basumatary UR, Mitra DK. Application of intuitionistic fuzzy optimization techniques to study multi-objective linear programming in agricultural production planning in Baksa district, Assam, India. Adv. Appl. Math. Sci... 2022; 21(6):3011-28.
17. Wang Y. Application of fuzzy linear programming model in agricultural economic management. Journal of Mathematics. 2022 Jan 24; 2022:1-3. https://doi.org/10.1155/2022/6089072
18. Yani, M., Asrol, M., Hambali, E., Papilo, P., Mursidah, S., & Marimin, M. (2022). An adaptive fuzzy multi-criteria model for sustainability assessment of sugarcane agroindustry supply chain. IEEE Access, 10, 5497-5517.
19. Hakmanage NM, Chandrasekara NV, Jayasundara M. Application of fuzzy goal programming model to assess optimal multi-crop cultivation planning. Agriculture and Natural Resources. 2022 Feb 25; 56(1):193-202.
20. Nawawi MK, Chaloob IZ, Ahmed JS, Ramli R, Sufahani SF. Maximization of Strategic Crops Production in Iraq with Fuzzy Goal Programming. Agricultural Research. 2022; 10(1):20-6. https://doi.org/10.13189/ujar.2022.100102
21. Mitlif RJ. An Efficient Algorithm for Fuzzy Linear Fractional Programming Problems via Ranking Function. Baghdad Science Journal. 2022 Feb 1;19(1):0071-. http://dx.doi.org/10.21123/bsj.2022.19.1.0071
22. Khan IU, Aftab M. Adaptive fuzzy dynamic programming (AFDP) technique for linear programming problems with fuzzy constraints. Soft Computing. 2023 Jun 20:1-9. https://doi.org/10.1007/s00500-023-08462-9
23. Fakhrahmad SM, Fakhrahmad SM, Soltani A, Hajipour K. Perceived environment and neighbourhood satisfaction: introducing a fuzzy modelling approach. Cities & Health. 2023 May 18:1-5. https://doi.org/10.1080/23748834.2023.2207924
24. Mahmoodirad A. A Novel Approach for Solving Linear Programming Problems with Intuitionistic Fuzzy Numbers. Fuzzy Optimization and Modeling Journal. 2023 Sep 1;4(3):1-5. https://doi.org/10.30495/fomj.2023.1995970.1113

25. Mahmud N, Alam MJ, Habib MA. Modelling of Activity Concentrations Using Machine Learning Technique. Procedia Computer Science. 2023 Jan 1; 220:462-9. https://doi.org/10.1016/j.procs.2023.03.059
26. Sharma, R., Tyagi, S. L. A New Algorithm to Solve Multi-objective Transportation Problem with Generalized Trapezoidal Fuzzy Numbers. Reliability: Theory & Applications, 2024 19(1 (77)), 531-543.
27. Yin, R., Jin, L., Fu, H., Fan, Y., Zhang, X., & Wang, L. (2024). Integrated multi-objective chance-constrained fuzzy interval linear programming model with principal component analysis for optimizing agricultural water resource management under uncertainties. Water Supply, 24(7), 2427-2450.
28. Pranav Dixit, Sohan Lal Tyagi. (2024). A Fuzzy Approach to Linear Programming in Agriculture Land Allocation. Journal of Computational Analysis and Applications (JOCAAA), 2024, 33(06), 760–766. https://eudoxuspress.com/index.php/pub/article/view/931
29. Lingo, 20.0, LINDO Inc. Ltd.