

Influence Of Porosity Effects And Thermal Radiation On Casson Fluid Flow Around A Stretching Cylinder

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Abstract

This work examines mass transfer processes in Sisko Ferro nanofluid across a permeable surface, with particular emphasis on numerical solutions of momentum and concentration equations using the R-K method. The core objective is to analyse the impact of various parameters on Schmidt number and concentration profiles. This also studies slip effects in MHD 3D non-newtonian Casson fluid flow across an exponentially stretched sheet. The Numerical approach of R-K method is applied on the flow using boundary layer condition and non-linear coupled equations regulating momentum, energy and mass transfer. Broadening its focus, the study investigates how Casson fluid flows along a vertically expanding cylinder are affected by natural convection, chemical processes, heat sources, and thermal radiation.

Keywords: Casson fluid, thermal radiation, chemical reaction, Schmidt number, dynamic model.

1. Introduction

The dynamics of fluid flow around objects has several applications in engineering and science, including as industrial processes and environmental systems, fluid behavior is impacted not only by inherent features, but also by extrinsic influences such as chemical reactions, heat impacts, and the existence of porous medium. Understanding and accurately forecasting the interactions of these events is critical for improving processes and creating efficient systems. This study investigates the complicated interaction of chemical reactions, temperature impacts, and porosity of Casson fluid with free convection flow around a stretched cylinder. Casson fluid, a non-Newtonian fluid model, shows shear-thinning property that is typical in many real-world applications. The stretched cylinder is a typical geometric form used in a various technical applications, including fiber spinning, wire coating, hydraulic linear motion, stent design for blood vessels.

Non-Newtonian fluids have recently gained significant technical application in industries. In reality, the popularity of convective boundary layer flows through Non-Newtonian fluids. [1] Examples of uses include drilling mud, hot rolling, cooling metallic and polymer plates, and so forth. Beevers and Craine investigated an incompressible binary combination of Newtonian fluids in 1982. [2] The study looks into radiative heat transfer in conjunction with laminar convection in a binary mixture under hydromagnetic influence. [3] An investigation is carried out using perturbation methods to (N.Sandeep, December 2018) evaluate time-dependent magnetohydrodynamic convection of heat and mass in a slip-flow boundary condition formed near a vertical permeable plate. [4] The study also investigates the dynamics of unsteady convection with radiative heat transfer and chemical reactions through a binary mixture passing through a flat porous plate. [5-6]. [7] Used a 2D model to investigate the MHD flow of the boundary condition with binary chemical reaction and energy of activation. Also conducted research on magneto-hydrodynamic convection flow through the boundary layer while taking into account of activated energy in heat-generating or heat absorbing reactions. [8]

[9] Demonstrated the effect of heat and mass transport on unstable rotational flows on binary mixture. Magnetohydrodynamic Casson fluids are used to study several elements of hydrodynamics and thermal processes. [10] The research delves into diverse phenomena within Hydrodynamics and thermal transport [11]. It explores the impact of momentum and thermal slip effects on stretching sheets, accounting for viscous dissipation [12]. Additionally, the study investigates the existence of multiple solution in transient mixed convection of MHD micropolar fluids over a sheet subjected to varying heat sources and sinks [13]. Another focus lies in applying cross-layer approximate computing techniques to advance from logic to architectural applications [14]. Furthermore, the study examines heat generation effects in three-dimensional MHD combined convection of Casson fluids over exponentially stretching sheets [15]. It also scrutinizes the unstable MHD radiative flow by heat transfer in nanofluids over stretched surface [16]. The investigation extends to analysing MHD thermal radiation effects on fluid flow and heat transport processes in nanofluids on stretched surfaces, aiming to elucidate the mixed influences over radiation and nanoparticle presence on fluid dynamics and heat transport mechanisms [17]. Additionally, the study applies the bivariate Spectral Collocation Quasi-Linearization Method to numerically explore Casson flow over nonlinear shrinking or stretching sheets. It illustrates unstable transfer mechanisms in MHD Casson flow near flat plates and vertical cones [18]. [19] investigated the MHD flow on nanofluid boundary condition on vertical sheet using Mathematica's ND Solve, accounting for activation energy and binary chemical processes. [20] An analysis is conducted to characterize a convective transport in a Casson fluid over a curved surface in the effects of thermal and chemical reactions. [21] explored how a binary chemical reaction affected the 2D MHD convective slip flow passing through a stretched sheet that radiates. Using an R-K based Shooting approach, [22] The study on a non-Newtonian Casson nanofluid with chemical interactions. [23] investigated the impact on magnetohydrodynamic (MHD) nanofluid circulation, considering buoyant effects. Additionally, [24] employed the R-K method to analyse unstable Casson fluid flow passing through stretched sheet. [25] utilized the perturbation method to study thermal and chemical reactions over an infinite vertical porous plate in casson flow by heat and mass transfer. Furthermore, [26] examined the effects of thermal radiation on chemically reacting Casson fluid flow over a plane, employing a shooting scheme and the 4th order R-K method. Moreover, [27] conducted a comprehensive thermal analysis of a non-Newtonian Casson fluid with magnetic interactions and chemical reaction. [28] applied the spectral quasilinearization technique (SQLM) to investigate mixed convective boundary layer condition to solve chemical reactions numerically. Lastly, [29] developed a homotopic analytic technique to explore three-dimensional Casson fluid flow under the impact of cross diffusion, advancing the understanding of complex fluid dynamics in boundary layers involving chemical reactions [30]. [31] studied the three-dimensional MHD cross-nanofluid flow when chemical reactions and activation energy were present. [32] Using modern methodologies and numerical simulations want to illuminate the complicated interplay between these events and give useful insights for improving system functionality and layout in a variety of engineering applications.

2. Literature Review

Prior analyses of Casson boundary layers has a magnetic fields, slip, and chemical reactions on flat or exponentially stretching sheets. Studies on cylindrical geometries demonstrate geometric thickening of the boundary layer compared with flat substrates. Porous-media

resistance has been shown to attenuate velocities, and radiative heat transfer to augment temperatures—yet their concurrent action with Casson rheology around stretching cylinders remains comparatively underexplored. The present work closes this gap by restricting attention to Casson fluids and isolating the roles of porosity and radiation

2. Governing problems

The influence of chemical processes under specified mass flux circumstances on mass transfer via a linearly expanding permeable surface inside a Sisko ferronanofluid is explored in this article. The ferrofluid model included in this work includes Brownian motion and thermophoresis, which are critical for describing the fluid's non-Newtonian behavior. Ferronanofluid, in particular, is a base fluid consisting of water and ethylene glycol, supplemented with ferroparticles such as Cobalt Ferrite nanoparticles (CoFe_2O_4), Manganese Zinc Ferrite ($(\text{Mn/Zn}) \text{Fe}_2\text{O}_4$) and Nickel Zinc Ferrite ($\text{NiZnFe}_2\text{O}_4$). The study describes the ferronanofluids' physical and thermal characteristics, which are essential to comprehending their behavior.

$$\tau = -PI + S, S = \left[a + b \left| \sqrt{\frac{1}{2} \text{tr}(A_1^2)} \right|^{n-1} \right] A_1 \quad (1)$$

Where b as consistency index, a is the viscosity parameter, n denotes power law index, A_1 as the kinematic tensor defined by $\text{grad}V + [\text{grad}V]^T$, I as identity tensor, P as pressure, S is the additional stress tensor, and τ denotes Cauchy stress tensor. The following is the problem's governing equation under the circumstances listed above.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (2)$$

In case u 's partial derivative with respect to x is denoted by $\partial u / \partial x$, then u 's partial derivative with respect to y is denoted by $\partial u / \partial y$.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{a}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{b}{\rho_{nf}} \frac{\partial}{\partial y} \left(-\frac{\partial u}{\partial y} \right)^n - \frac{\sigma B_0^2 u}{\rho_{nf}} - \frac{v_{nf}}{K_p} u \quad (3)$$

$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$ represents the convective acceleration of the fluid velocity u in x and v in y directions, a , b and n are constants

$\frac{a}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2}$ represents diffusion of velocity u in y direction, ρ_{nf} is nanofluid density. $-\frac{\sigma B_0^2 u}{\rho_{nf}}$: B_0 represents the magnetic field strength with velocity u , where σ is electrical conductivity.

$-\frac{v_{nf}}{K_p} u$: Represents the force due to viscous dissipation, where v_{nf} is the kinematic viscosity of nanofluid and K_p is the permeability of porous medium

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial y} - Q' \frac{\partial T}{\partial y} + \frac{a}{(\rho C_p)_{nf}} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{b}{(\rho C_p)_{nf}} \left(-\frac{\partial u}{\partial y} \right)^{n+1} + \frac{(\rho C_p)_p}{(\rho C_p)_f} \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (4)$$

The term $u \frac{\partial T}{\partial x}$ represents convective heat transport with respect to x .

The term $v \frac{\partial T}{\partial y}$ represents convective heat transport with respect to y .

$\alpha_{nf} \frac{\partial^2 T}{\partial y^2}$ describes diffusion of temperature T with respect to y , influenced by the thermal diffusivity α_{nf} .

$-\frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial y}$ describes radiative heat flux term with respect to y.

$-Q' \frac{\partial T}{\partial y}$ denotes advective heat transfer, where it is internal heat source/sink or heat absorption effect

$\frac{a}{(\rho C_p)_{nf}} \left(\frac{\partial u}{\partial y}\right)^2$ reflects the buoyancy effect, with u being the velocity component in the y direction.

$\frac{b}{(\rho C_p)_{nf}} - \left(\frac{\partial u}{\partial y}\right)^{n+1}$ represents shear-induced mixing, where n is an exponent typically related to the rheological behavior of the fluid.

$\frac{(\rho C_p)_p}{(\rho C_p)_{nf}} D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + D_T \frac{T_\infty}{T} \frac{\partial^2 T}{\partial y^2}$ accounts for interactions between phases if T is influenced by another phase C. Here, $(\rho C_p)_p$, $(\rho C_p)_{nf}$, D_B and D_T are diffusivities related to mass and thermal transfer between phase and it is nanoparticle heat capacity by involving Brownian and Thermophoresis effects

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_1 [C - C_\infty] \quad (5)$$

$u \frac{\partial C}{\partial x}$: Describes the advection term of concentration C with respect to x

$v \frac{\partial C}{\partial y}$: Represents the advection term of concentration C with respect to y

$D_B \frac{\partial^2 C}{\partial y^2}$: Describes diffusion of concentration C with respect to y, where D_B diffusion coefficient.

$\frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}$: Represents thermophoresis diffusion term, where D_T cross-diffusion coefficient between concentration C and temperature T, and T_∞ temperature of the surrounding medium.

$-k_1 [C - C_\infty]$: Describes the chemical reaction term, where k_1 chemical reaction constant and C_∞ concentration in the surrounding medium. This term accounts for the change in concentration C due to chemical reactions.

The conditions at the boundaries

$$u(x, y) = U = cx^s \quad (6)$$

$$v(x, y) = -V_0 \quad (7)$$

$$k_f \frac{\partial T}{\partial y} = q_w = E_0 \left(\frac{x}{L}\right)^m \text{ at } y = 0 \quad (8)$$

$$D_B \frac{\partial C}{\partial y} = m_w = E_0 \left(\frac{x}{L}\right)^m \text{ at } y = 0 \quad (9)$$

$$u = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (10)$$

The variables 's' and 'm' represent the power-law velocity and surface temperature change, respectively, and the constant 'c', characteristic length 'L,' and surface mass flow, m_w .

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, v_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_p \quad (11)$$

- μ_{nf} : effective viscosity of mixture.
- μ_f : viscosity of fluid phase.
- ϕ (phi): volume fraction of dispersed phase (such as particles or bubbles).
- v_{nf} : kinematic viscosity of mixture.

- ρ_{nf} : density of mixture.
- ρ_f : density of fluid phase.
- ρ_p : density of dispersed phase.

The following similarity transformations.

$Q' = -dQf(\eta)$ denotes the linear function of the temperature field's volumetric rate.

$$d = UR_{eb}^{\frac{-1}{n+1}}, \psi = UxR_{eb}^{\frac{-1}{n+1}}f(\eta), \eta = \frac{y}{x}R_{eb}^{\frac{1}{n+1}}, \quad (12)$$

$$u = f'(\eta)U \quad (13)$$

η is a dimensionless parameter that typically represents a normalized distance or length in the flow.

$f'(\eta)$ is a function that describes how the velocity u varies with η .

$f'(\eta)$ encapsulates the specific velocity profile shape characteristic of the flow. Common forms include linear profiles ($f'(\eta) = 1$), parabolic profiles ($f'(\eta) = 1 - \eta^2$), or other profiles depending on the flow dynamics.

$$v = -UR_{eb}^{\frac{-1}{n+1}} \frac{1}{n+1} [\{s(2n-1) + 1\}f(\eta) + \{s(2-n) - 1\}\eta f'(\eta)] \quad (14)$$

The equation describes how v , the velocity component, depends on $U, R_{eb}, f(\eta)$, and $f'(\eta)$. The term involving s, n , and coefficients adjusts the contribution of $f(\eta)$ and $\eta f'(\eta)$ to the overall velocity v .

The negative sign indicates the direction of v relative to U and R_{eb} .

$$T - T_{\infty} = \frac{E_0}{k_f} \left(\frac{x}{L}\right)^m \times R_{eb}^{\frac{-1}{n+1}} g(\eta)$$

The equation describes how the temperature difference $T - T_{\infty}$ depends on various factors. $\frac{E_1}{k_f} \frac{x^m}{L}$ represents the heat generation or source term scaled by the thermal conductivity and characteristic length scale.

$xR_{eb}^{\frac{-1}{n+1}}$ adjusts the contribution based on the Reynolds number and the exponent n .

$g(\eta)$ modulates how temperature varies with the dimensionless parameter η , capturing the spatial variation of temperature due to the flow dynamics.

$$C - C_{\infty} = \frac{E_0}{D_B} \left(\frac{x}{L}\right)^m \times R_{eb}^{\frac{-1}{n+1}} h(\eta) \quad (15)$$

The dimensionless stream function is denoted by Ψ , and the similarity variable, η , is contingent on both x and y .

Equation (5) may be substituted into equations (2), (3), and (4) to get

$$\begin{aligned} Af'''' + n(-f'')^{n-1}f'''' + \phi_1 \left(\frac{s(2n-1)+1}{n+1} ff'' - sf'^2 \right) - \left(M + \frac{1}{Rp\phi_2} \right) f' &= 0 \\ g'' + D \left(\frac{s(2n-1)+1}{n+1} + Q \right) fg' - (m+1)Df'g + DN_b g'h' + N_t Dg''^2 &= \\ - \frac{E_c D}{\phi_3} (A(f'')^2 + (-f'')^{n+1}) & \end{aligned} \quad (16)$$

$$h'' + Le \ Pr \frac{s(2n-1)+1}{n+1} fh' - (m+1)LePr_r f'h - \gamma LePr h + \frac{N_t}{Nb} g'' = 0 \quad (17)$$

With boundary conditions

$$\begin{aligned} f'(\eta) &= 1, f(\eta) = \lambda \\ g'(\eta) &= -1, h'(\eta) = -1, \\ \text{at } \eta &= 0 \end{aligned} \quad (18)$$

Where $\phi_1 = 1 - \phi + \phi \frac{\rho_s}{\rho_f}$, $\phi_2 = (1 - \phi)^{2.5}$, $A = \frac{2}{R_{ea}^{n+1}}$ is material parameter of Sisko fluid $R_{ea} = \frac{\rho_f x U}{a}$ and $R_{eb} = \frac{\rho_f x U^{2-n}}{b}$ are local Reynolds number, $\lambda = \frac{V_0}{UR_{eb}^{n+1}}$ is suction parameter, $R_p = \frac{K_p U}{v_f x}$ is permeability parameter, $M = \frac{\sigma B_0^2 x}{\rho_f U}$ is magnetic parameter, $P_r = \frac{x U R_{eb}^{n+1}}{\alpha_f}$ is Prandtl number, $\gamma = \frac{k_1 x}{U}$ (chemical reaction parameter), $L_e = \frac{\alpha_f}{D_B}$ (Lewis number), $N_b = \frac{(\rho C_p)_p}{(\rho C_p)_f} \frac{D_B (c_f - c_\infty)}{\alpha_f}$ is Brownian motion parameter, $N_t = \frac{(\rho C_p)_p}{(\rho C_p)_f} \frac{D_T (T_f - T_\infty)}{T_\infty \alpha_f}$ is thermophoresis parameter.

Sherwood number is defined from the relation

$$N_{u_x} = \frac{x m_w}{D_B (C_f - C_\infty)} \text{ at } y = 0$$

Using equations (4), (5) and (9) the dimensionless Sherwood number can be written as

$$R_{eb}^{n+1} S_{h_x} = \frac{1}{h(0)} \quad (19)$$

3. Extending in 3 D MHD Non-Newtonian Casson fluid flow:

Consider Three- dimensional MHD flow of non- Newtonian Casson fluid induced by an exponential stretching sheet with existing of Binary chemical reaction and Activation energy. The Cartesian coordinate axes are (x, y, z) considered with corresponding velocities (u, v, w) and the sheet has stretched along xy plane at $z = 0$. The uniform magnetic field B_0 is applied towards the positive direction of z axis. Assume that the exponential form of stretching velocities of the sheet are $U_w = U_0 e^{\frac{x+y}{L}}$ and $V_w = V_0 e^{\frac{x+y}{L}}$ respectively. The Modified Arrhenius function $\left(\frac{T}{T_\infty}\right)^n e^{\left(-\frac{E_0}{kT}\right)}$ is taken into account. Considering the following formulation of the governing equations for the non-Newtonian Casson fluid flow may be used:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (20)$$

Momentum Equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \vartheta \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u \quad (21)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \vartheta \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v \quad (22)$$

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_m \frac{\partial^2 T}{\partial z^2} + \frac{Q}{\rho C_p} (T - T_\infty) \quad (23)$$

Concentration Equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} - k_r^2 \left(\frac{T}{T_\infty} \right)^n \exp \left(\frac{-E_a}{kT} \right) (C - C_\infty) \quad (24)$$

Subject to following boundary conditions

$$\left. \begin{aligned} u = U_w = U_0 e^{\frac{x+y}{2L}}, v = V_w = V_0 e^{\frac{x+y}{2L}}, w = 0, \\ T = T_w = T_\infty + T_0 e^{A\left(\frac{x+y}{2L}\right)} + L_1 \frac{\partial T}{\partial z}, C = C_w = C_\infty + C_0 e^{B\left(\frac{x+y}{2L}\right)} + L_2 \frac{\partial C}{\partial z} \end{aligned} \right\} \text{ at } z = 0 \quad (25)$$

$$u = 0, v = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z \rightarrow \infty$$

The following similarity transformations have been used to convert the previously described partial differential equations (1)–(5) into a set of ordinary differential equations: dimensionless temperature $\theta(\eta)$, dimensionless concentration $\phi(\eta)$, dimensionless stream functions $f(\eta)$, and

$$\eta = \left(\frac{U_0}{2vL} \right)^{1/2} e^{\frac{x+y}{2L}} z, u = U_0 e^{\frac{x+y}{2L}} f'(\eta), v = V_0 e^{\frac{x+y}{2L}} g'(\eta) g(\eta) \quad (26)$$

$$w = - \left(\frac{vU_0}{2L} \right)^{1/2} e^{\frac{x+y}{2L}} (f + \eta f' + g + \eta g'), \theta(\eta) = \frac{T - T_\infty}{T_0 e^{\frac{A(x+y)}{2L}}}, \phi(\eta) = \frac{C - C_\infty}{C_0 e^{\frac{B(x+y)}{2L}}} \quad (27)$$

The governing boundary layer equations (2)–(5) becomes

$$\left(1 + \frac{1}{\beta} \right) f''' + 2(f' + g')f' + (f + g)f'' - M^2 f' = 0 \quad (28)$$

$$\left(1 + \frac{1}{\beta} \right) g''' + 2(f' + g')g' + (f + g)g'' - M^2 g' = 0 \quad (29)$$

$$\theta'' + Pr [(f + g)\theta' - A(f' + g')\theta + S\theta] = 0 \quad (30)$$

$$\phi'' + Sc \left[(f + g)\phi' - B(f' + g')\phi - \sigma(1 + \delta\theta)^n \exp \left(\frac{-E}{(1 + \delta\theta)} \right) \phi \right] = 0 \quad (31)$$

The relevant boundary conditions are

$$\begin{aligned} f(0) = 0, g(0) = 0, f'(0) = 1, g'(0) = \alpha, \theta(0) \\ = 1 + \gamma_1 \theta'(0), \phi(0) = 1 + \gamma_2 \phi'(0) \text{ at } \eta = 0 \end{aligned} \quad (32)$$

The non-dimensional parameters σ , M , E , δ , Sc , Pr , B , A , β , S , γ_2 , and γ_1 represent the rate constant of chemical reactions, magnetic parameter, activation energy, temperature relative parameter, Schmidt number, Prandtl number, concentration exponent, temperature exponent, casson fluid parameter, heat source parameter, concentration slip parameter, and temperature slip parameter, respectively.

$$\phi(0) = 1 + \gamma_2 \phi'(0) \quad (33)$$

$\phi(0)$ represents the value of the function $\phi(\eta)$ evaluated at $\eta=0$. In other words, it gives us the value of ϕ at the starting point or boundary of the interval or domain over which ϕ is defined. $1 + \gamma_2 \phi'(0)$: This expression consists of: **1**: A constant term. γ_2 : A coefficient that could represent a constant parameter or a coefficient associated with the derivative $\phi'(\eta)$. The derivative of $\phi(\eta)$ evaluated at $\eta=0$. The equation implies that the value of $\phi(0)$ is determined by adding 1 to γ_2 times the value of $\phi'(\eta)$ at $\eta=0$. This relationship could arise in various contexts, such as in boundary value problems for differential equations, where both the function and its derivative are specified or related at a particular boundary point.

4. Analysis

Table 1 and figure 1 presents the results of different approximation orders for three functions at $x=0$. These trends reflect how the functions' behaviors adjust with more refined approximations. Overall, the data reveals how the curvature and slopes of the functions θ , and ϕ adjust with increasing approximation orders. The trends show subtle variations in the functions' behaviors, reflecting the impact of more precise approximations on their derivatives at $x=0$.

Table 1 Convergence for different order of values of homotopy solution

ϕ			
Order of approximation	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	0.8265	0.1456	0.6485
5	0.8214	0.1659	0.5784
12	0.8184	0.1714	0.4945
20	0.8182	0.1782	0.4932
28	0.8179	0.1724	0.4989
35	0.8147	0.1704	0.4999
40	0.8121	0.1701	0.5012

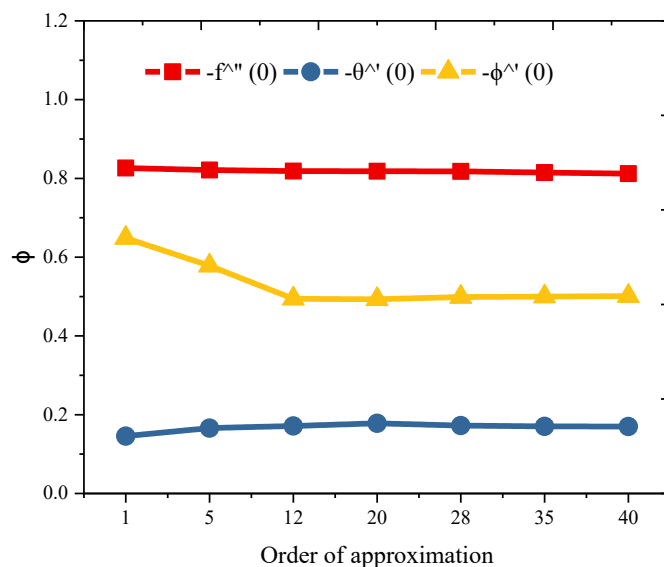


Figure 1: Homotopy solutions for Convergence pattern

This part is presented to draw researchers' attention by examining how chosen parameters affect transport equations. the impact of several factors, including chemical reaction rate constant(σ^2), temperature relative parameter (δ), and activation energy (E).The computed

findings are described and the fluctuations in the temperature, concentration, and velocity fields are shown graphically in Figures

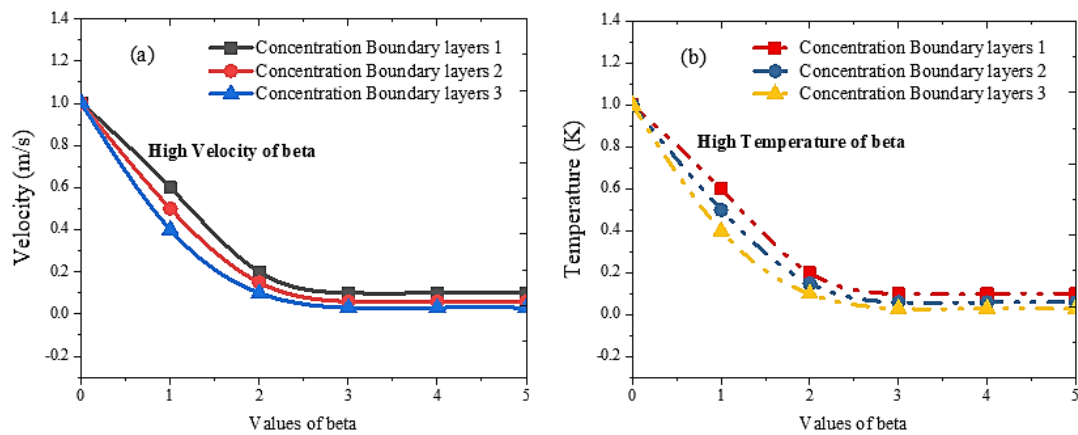
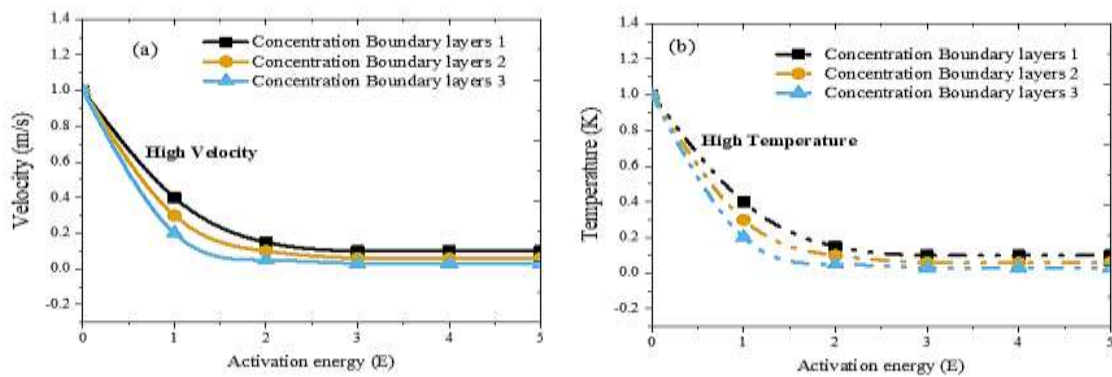


Figure 2 (a) and (b) temperature profile of different values of beta



Figures 3 (a) and (b) were made to look into how activation energy (E) affected the temperature, concentration, and velocity profiles for Casson fluid.

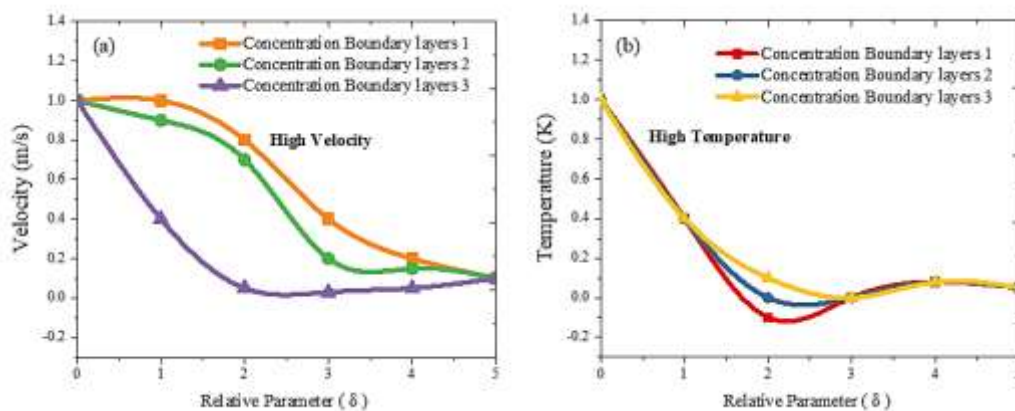


Figure 4 (a) and (b) shows temperature relative parameter (δ) in concentration boundary layers

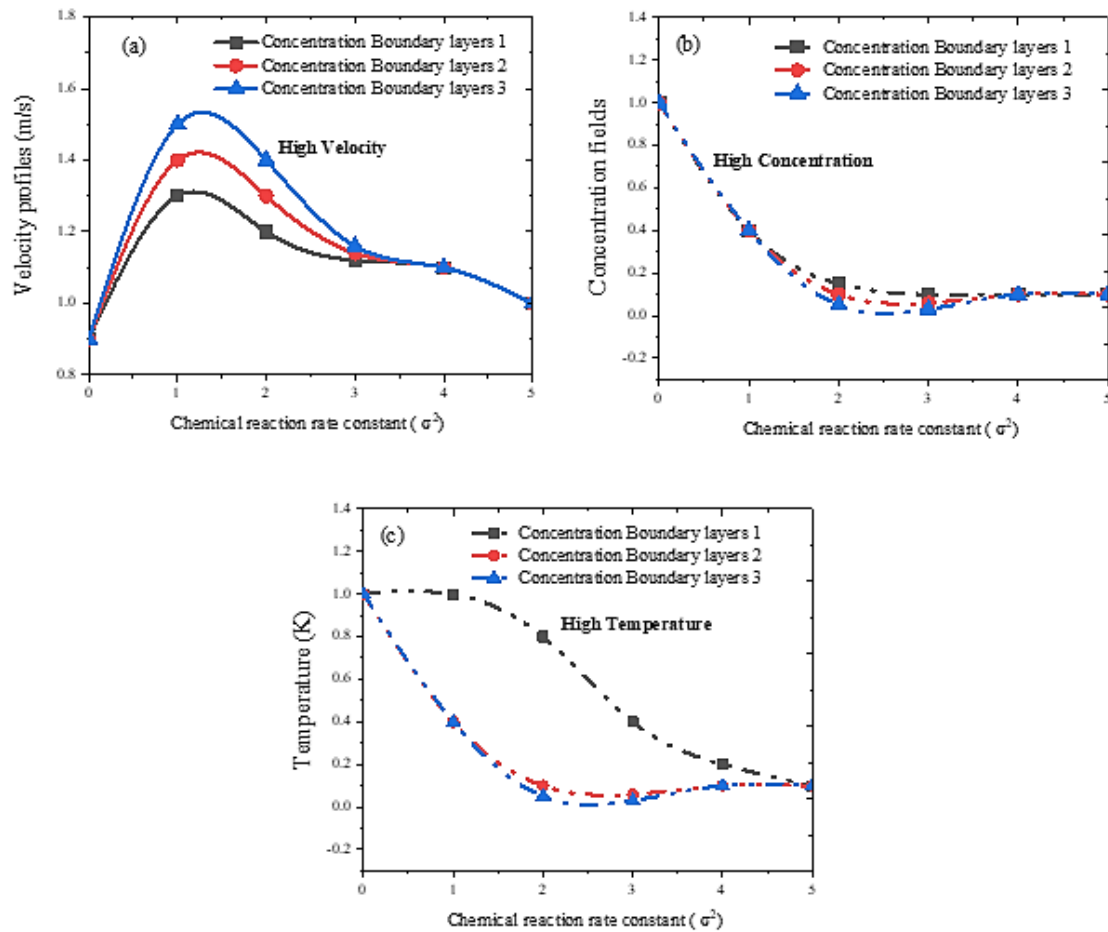
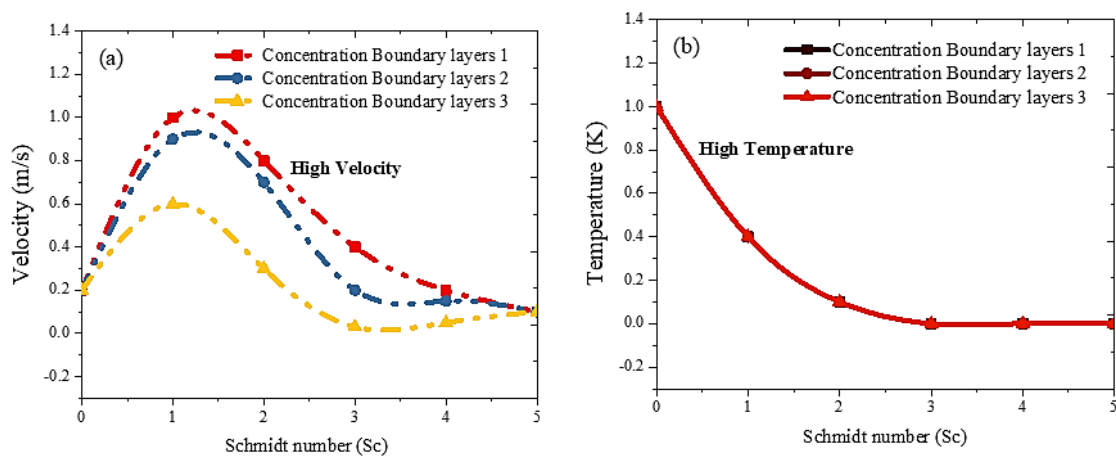


Figure 5 (a) (b) and (c) Chemical reaction rate constant (σ^2) on velocity, temperature and Concentration fields



Figures 6 (a) and (b) depict behavior of Schmidt number Sc on temperature, velocity, and concentration profiles against the variable η in the boundary layer.

CONCLUSION

The outcome has higher estimations in Schmidt number and concentration profile diminished and study can be extending the present work by considering magnetic field influence (magnetohydrodynamics) in physical and chemical reactions. The inclusion of nanoparticle suspensions (nanofluids) or hybrid nanofluids could further enhance characteristics in Casson fluids. Finally, experimental validation of the theoretical results would strengthen in ambient engineering, particularly in biomedical processes, polymer processing, and industrial cooling technologies

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