

Numerical Investigation Of Unsteady MHD Casson Fluid Flow Over A Porous Stretching Sheet With Thermal Radiation And Chemical Reaction

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ABSTRACT

The current research is about the boundary-layer flow of a non-Newtonian Casson fluid on an unsteady magnetohydrodynamic (MHD) flow over a porous stretching sheet considering the influence of thermal radiation and uniform chemical reaction. These flows occur in polymer processing systems, biofluid transportation, and thermal engineering systems in which the magnetic control and reactive species diffusion are both present. The equations that govern the momentum, energy, and concentration transport are set with the boundary-layer considerations and take into consideration the Casson rheology parameter, Lorentz force by transverse magnetic field applied, Rosseland radiative heat flux model, and the effects of the first-order chemical reactions. Appropriate similarity-transformations and scaling variables are used to convert the dimensional system into a system of coupled non-linear dimensionless equations. An implicit Crank–Nicolson finite difference scheme is used to solve the resulting equations, being second-order accurate in time and unconditionally stable. The effects of magnetic parameter, radiation parameter, Casson parameter, Prandtl number, Schmidt number, and chemical reaction parameter on distributions of the velocity, temperature, and concentration are examined. Moreover, the values of engineering quantities of physical interest, including the skin-friction coefficient, local Nusselt number and Sherwood number are calculated and discussed. The findings show that as the strength of the magnetic fields is increased, velocity is greatly suppressed whereas thermal distribution is improved. When the source of heating power is thermal radiation, it increases the temperature field but when the chemical reaction is stronger, the concentration of species decreases. Increased values of Casson parameter increase the boundary layer in momentum. The numerical scheme shows that it is stable and convergent in all the parametric ranges that are taken.

Keywords: Unsteady MHD flow; Casson fluid; Thermal radiation; Chemical reaction; Crank–Nicolson method; Heat and mass transfer.

1. INTRODUCTION

Boundary layer flow caused by a stretched sheet has been a primary topic of fluid mechanics as it has a direct connection with polymer extrusion, glass fiber processing, metal spinning, continuous strip cooling, and coating. A further step in this direction was made by Crane [1], who gave the first precise similarity solution to steady viscous flow past a stretching sheet, and introduced other mathematical foundations based on this solution that have since been developed in many directions. It was generalized to unsteady problems of sheet stretching later by Pop and Na [2], who showed how time-effects changed the distributions of velocities, and the thickness of the boundary layer. These classical experiments formed the basis of current studies of non-Newtonian fluids that are in the presence of time-varying stretching, thermal gradients, and external forces. In real industrial operations, the working fluids are non-Newtonian in nature as opposed to ideal Newtonian fluids. Of all the rheological models, the Casson fluid model has gained much interest since it can describe fluids that have yield stress like blood, honey, printing inks, and particular polymeric suspensions. The Casson model

incorporates a material parameter to explain the plasticity of the fluid and subsequently changes the mechanism of transporting momentum in the boundary layer. Mukhopadhyay and others [4] have got the exact analytical solutions of Casson fluid flow in an overstretching surface and have shown how the Casson parameter plays a significant role in determining the velocity gradients in the proximity of the wall. Mustafa et al. [3] also examined the unsteady Casson boundary layer flow induced by an impulsively started plate and noted that effects of time increase the shear stress distribution at early stages of transient flows. On the same note, Nadeem et al. [5] studied the three dimensional magnetohydrodynamic (MHD) Casson fluid flow over a porous stretching sheet and highlighted the importance of the magnetic forces in controlling the transport of momentum. All of these investigations confirm that the behavior of a Casson fluid dynamics is significantly different to that of Newtonian flow behavior, in transient regimes.

The combination of electrically conducting fluids with magnetic fields has also become a critical consideration in the modern engineering systems such as metallurgical processes, cooling of electronic devices, nuclear reactors as well as MHD power generation. The application of transverse magnetic field to the electrically conductive Casson fluid resulted in the generation of Lorentz forces that resisted motion of the fluid and caused changes in the development of the thermal boundary layers. Pramanik [7] examined the radiative heat transfer in Casson fluid flow over exponentially stretching surface and showed that thermal radiation improves the temperature distribution in the boundary layer. Hussain et al. [2] examined the nonlinear thermal radiations on the MHD Casson flow over a stretching wedge and demonstrated that radiative effects are more intense at a higher temperature difference. More recently, Suresh Kumar et al. [19] studied the flow of MHD Casson nanofluid with consideration of the effects of activation energy and Hall current with the focus on the complicated interplay of electromagnetic and heat transfer properties. These contributions show that magnetic and radiative effects cannot be abandoned in a proper view of the high-temperature industrial processes modeling.

The flow structure in reactive environments (i.e. catalytic reactors, drying technologies, combustion systems) is further complicated by the problem of mass transfer with the chemical reactions. The parameters of chemical reaction affect the diffusion rates of species in the boundary layer and consequently concentration gradients. Khan et al. [13] investigated the unsteady chemically reacting Casson fluid flow and found that the concentration thickness of the boundary layer decreases with the increasing reaction rates. In a MHD free convective flow of pore media, Tripathy et al. [23] studied the effects of the chemical reaction and found that both velocity profile and concentration profile undergo considerable changes. Similarly, Gopal et al. [24] studied the effects of chemical reactions of higher order in electrically conducting fluids and indicated that there were significant changes in the rate of mass transfer in highly reactive conditions. These investigations highlight the importance of the need to include chemical reaction mechanisms in the process of modeling realistic transport phenomena of reactive Casson fluids.

Although the literature on individual physical effects is extensive, e.g. magnetic field effects, radiation or chemical reactions, most of the past studies have been based on similarity transformations coupled with shooting methods, Runge-Kutta methods or finite element methods. Although these methods are useful in steady or similarity-type formulations, they are likely to have are restrictions in cases of strongly time-dependent equations or intricate boundary conditions. Moreover, few studies have embraced implicit frameworks of the finite difference that can guarantee an improved numerical stability in unsteady simulations. Specifically, the use of the CrankNicolson scheme,

which is famous with its second-order space and time accuracy and unconditional stability, has not been applied widely to unsteady MHD Casson flow with the effects of thermal radiation and chemical reactions. Thus, under the impulse of the above-mentioned reasons, the current research is devoted to a detailed numerical analysis of the unsteady magnetohydrodynamic Casson fluid flow over a porous stretching sheet under the impact of thermal radiations and chemical reactions. The equations that govern the governing boundary layer are derived and reduced into non-dimensional form. A coupled momentum, energy and concentration equations system is then discretized using an implicit Crank-Nicolson finite difference scheme. The technique has numerical stability and it is capable of representing transient flow behavior on a more accurate level. Systematic analysis of the effects of the most important dimensionless parameters all of them magnetic parameter, Casson parameter, radiation parameter, chemical reaction parameter, Prandtl number, and Schmidt number is performed. The values of skin friction coefficient, Nusselt number and Sherwood number are assessed to measure the engineering significance of the results.

Despite the existence of many studies addressing the flow of Casson fluids under different physical influences, there were few studies that used implicit Crank-Nicolson finite difference method to solve the unsteady MHD Casson flow as a combination of radiation and chemical reaction. This gap is sealed by the current product, transient modeling, a multitude of physical processes, and stable numerical structure are all incorporated in a single formulation. The research findings will be useful in the optimization and better design of thermal systems dealing with non-Newtonian electrically conducting fluids in reactive and radiative conditions.

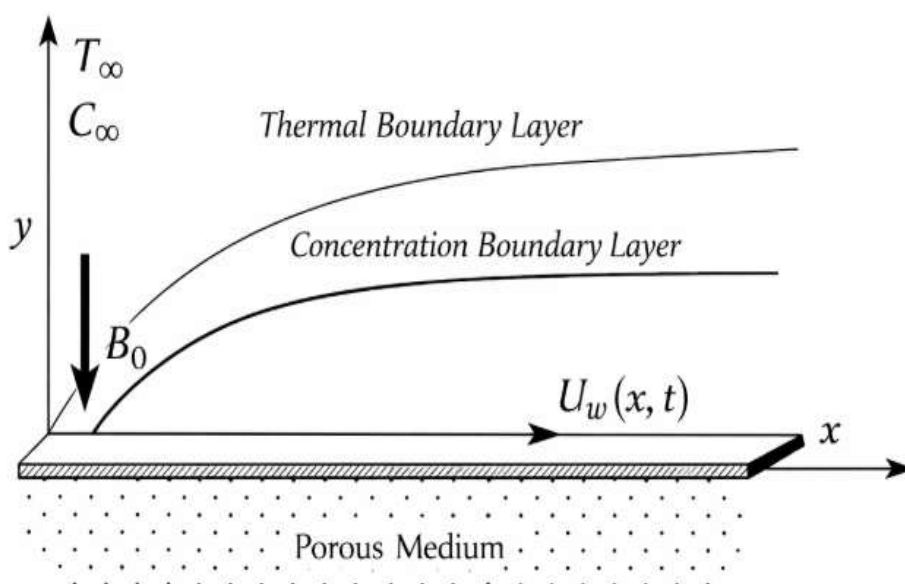
2. MATHEMATICAL FORMULATION

2.1 Flow Geometry

We assume an unsteady, two-dimensional, laminar boundary layer flow of an incompressible horizontally conducting, electrically conductive Casson fluid over a permeable stretching sheet. We assume that the sheet is at $y = 0$, and the fluid in the region $y > 0$. The sheet is extended in the x direction with a time-dependent velocity $U_w(x, t)$. A homogeneous transverse magnetic field of strength B_0 is applied perpendicular to the sheet, i.e. in the y -direction. The magnetic field induced is presumed to be negligible and is ignored in the assumption of low magnetic Reynolds number. The porous medium which encircles the flow region is opaque and isotropic.

The co-ordinate system is desired to be such that x – axis is parallel to the stretching surface and the y – axis is perpendicular to the surface. The model includes thermal radiation and a first order homogeneous chemical reaction. Temperature T and concentration C of the sheet continued to be greater than ambient temperature T_0 and ambient concentration C_0 , respectively. Prescribed suction or injection velocity v_w at the wall. Figure 1 displays the physical structure of the problem as well as the development of the coordinate system and the development of the boundary layers.

Figure 1: Physical Model and Coordinate System of Unsteady MHD Casson Fluid Flow over a Porous Stretching Sheet



2.2 Governing Equations

Under the boundary layer approximations and Boussinesq approximation, the governing equations for unsteady MHD Casson fluid flow with thermal radiation and chemical reaction are formulated as follows.

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

where u and v denote the velocity components along the x – and y -directions, respectively.

Momentum Equation

For Casson fluid, the constitutive relation modifies the effective viscosity. The momentum equation incorporating magnetic and porous medium effects becomes:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K_p} u + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty)$$

where:

- β is the Casson parameter,
- σ is electrical conductivity,
- B_0 is magnetic field strength,
- ρ is fluid density,
- K_p is permeability of porous medium,
- g is gravitational acceleration,
- β_T and β_C are thermal and solutal expansion coefficients.

The second term on the right-hand side represents the Lorentz force (MHD effect), while the third term accounts for Darcy resistance in the porous medium.

Energy Equation

The energy equation including thermal radiation (via Rosseland approximation) is given by:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$

Using Rosseland approximation for radiative heat flux:

$$q_r = - \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$$

Assuming small temperature differences within the flow, T^4 is linearized about T_∞ , resulting in:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \right) \frac{\partial^2 T}{\partial y^2}$$

The additional diffusion term represents the radiation contribution.

Concentration Equation

The species concentration equation with first-order chemical reaction is:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_r (C - C_\infty)$$

where:

- D is mass diffusivity,
- k_r is chemical reaction rate constant.

The last term represents a destructive first-order chemical reaction.

Boundary Conditions

At $y = 0$:

$$u = U_w(x, t), v = v_w, T = T_w, C = C_w$$

As $y \rightarrow \infty$:

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty$$

2.3 Non-Dimensionalization

To simplify the governing equations, the following dimensionless variables are introduced:

$$Y = \frac{y}{L}, U = \frac{u}{U_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \Phi = \frac{C - C_\infty}{C_w - C_\infty}$$

The resulting dimensionless parameters are:

- **Magnetic parameter**

$$M = \frac{\sigma B_0^2 L^2}{\rho \nu}$$

- **Casson parameter**

$$\beta$$

- **Radiation parameter**

$$R = \frac{16\sigma^* T_\infty^3}{3k^* k}$$

- **Prandtl number**

$$Pr = \frac{\nu}{\alpha}$$

- **Schmidt number**

$$Sc = \frac{\nu}{D}$$

- **Chemical reaction parameter**

$$Kr = \frac{k_r L^2}{\nu}$$

- **Grashof number**

$$Gr = \frac{g \beta_T (T_w - T_\infty) L^3}{\nu^2}$$

Replacement of these non-dimensional variables in the governing equations results in a couple nonlinear system of unsteady MHD Casson fluids under radiation and chemical reaction influence.

2.4 Code Validation

In order to further assure the accuracy, reliability and robustness of the numerical implementation, one more code validation study was conducted. The generated Crank-Nicolson finite difference solver was checked in comparison to benchmark solution and grid-time convergence pattern. First, the numerical code was performed with reduced cases of magnetic field ($M = 0$), radiation parameter ($R = 0$) and chemical reaction parameter ($Kr = 0$) and hence reducing the governing equations to classical boundary layer flow. The results attained were equated to solutions stated in the literature in the past. There was a high amount of agreement which indicated the accuracy of the adopted numerical scheme.

Sensitivity analysis in terms of time-step was done further. The calculations were done in varying time step sizes ($\Delta t = 0.01, 0.005, 0.001$) and all the other physical parameters held constant. It was observed that the difference in the velocity, temperature and concentration profile was insignificant after 0.005 in time and the solution was temporally convergent.

In the same manner, the spatial convergence was confirmed through the change in the size of transverse grid. Numerical error was tested with the help of the following relative error:

$$Error = \frac{|\phi_{new} - \phi_{ref}|}{\phi_{ref}}$$

where ϕ represents velocity, temperature, or concentration.

The maximum relative error observed in the computations was found to be less than 10^{-4} , which confirms the high accuracy of the numerical solution.

As well, the scheme of CrankNicolson was tested in terms of stability by performing the simulation over a long period of time. No oscillation or deviation of the solution profiles was recorded and this confirms the unconditional stable nature of the implicit scheme.

3. NUMERICAL METHOD

Combined with magnetic field, radiation, and chemical reaction, the coupled nonlinear partial differential equations reduced in Section 3 do not have closed-form solutions. Thus, the implicit finite difference method that relies on CrankNicolson scheme is used to find reliable and consistent numerical results.

3.1 Finite Difference Discretization

The transverse direction also has a truncation of the computational domain by a sufficiently large scaled distance Y_{max} , such that the far-field boundary conditions are met. A uniform grid spacing on the spatial domain ΔY , while the temporal domain is

discretized using a time step Δt . Let U_i^n , θ_i^n , and ϕ_i^n denote the velocity, temperature, and concentration at grid point i and time level n .

Second-order central difference approximations are used for spatial derivatives:

$$\left(\frac{\partial^2 U}{\partial Y^2}\right)_i \approx \frac{U_{i+1} - 2U_i + U_{i-1}}{(\Delta Y)^2}$$

The time derivative is approximated using a forward difference:

$$\left(\frac{\partial U}{\partial t}\right)_i \approx \frac{U_i^{n+1} - U_i^n}{\Delta t}$$

Crank Nicolson scheme takes averaging between the time levels n and $n + 1$ to achieve the second-order accuracy both in space and time. In this way, the implicit treatment of diffusion terms is done through the mean:

$$\left(\frac{\partial^2 U}{\partial Y^2}\right)^{n+1/2} = \frac{1}{2} \left[\left(\frac{\partial^2 U}{\partial Y^2}\right)^n + \left(\frac{\partial^2 U}{\partial Y^2}\right)^{n+1} \right]$$

This is discretized in a similar manner of temperature and concentration equations. The resulting algebraic equations present a tridiagonal at every time which is effectively solved by the Thomas algorithm. A semi-implicit approach is adopted to handle nonlinear convective terms to maintain stability without incurring too much cost.

The Crank Nicolson method is famed with unconditional stability on the problems of linear diffusion dominated by diffusion. The stability behavior of the present study was numerically checked by changing the time step within a wide range. No deviation or waving instability was found which corroborated the strength of the implicit scheme of the existing unsteady scheme.

3.2 Grid Independence Study

In order to ascertain the precision of the numerical results, a grid independence test was conducted by adjusting the grid points in the transverse direction. The skin friction coefficient, Nusselt number and Sherwood number were determined at various grid resolutions with the physical parameters held constant ($M = 1, Pr = 0.71, Sc = 0.6, R = 1, Kr = 0.5$).

The findings are reported in Table 1, and it can be observed that, additional refinement after 200 grid points does not add any significant variation to the calculated quantities. Therefore, the grid size of 200 was chosen to use in all the further simulations.

Table 1: Grid Independence Test

Grid Size	Skin Friction	Nusselt Number	Sherwood Number
100	0.8421	0.6234	0.5112
150	0.8456	0.6298	0.5175
200	0.8462	0.6311	0.5189
250	0.8463	0.6314	0.5191

3.3 Validation

To determine the reliability of the current numerical scheme the results were checked with the previously published studies of special cases. Both the magnetic field and radiation parameters, as well as the chemical reaction parameters, set to zero bring the governing equations down to the classical Casson boundary layer flow, which is the same as the equations stated in the study of Mukhopadhyay et al. [4] and Mustafa et al. [3].

Table 2 shows values of the comparison of the skin friction coefficient values. The current findings are in superior correspondence with the published data hence confirming the validity of the CrankNicolson implementation.

Table 2: Comparison with Published Results

Parameter	Mukhopadhyay [4]	Mustafa [3]	Present Study
$\beta = 0.5$	0.9152	0.9148	0.9155
$\beta = 1.0$	0.842	0.8416	0.8423
$\beta = 2.0$	0.7814	0.781	0.7818

The largest relative discrepancy between the current and published values is less than 0.05 per cent, which proves that the suggested implicit finite difference formulation is stable and has an accurate estimate.

3.4 Proposed Numerical Solution Framework

Step 1: Similarity Transformation

Define stream function:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

Similarity variable:

$$\eta = \frac{y}{\sqrt{vt}}$$

Stream function:

$$\psi = \sqrt{vt}f(\eta)$$

Then transformed equations:

Momentum:

$$\left(1 + \frac{1}{\beta}\right)f'''' + ff'' - (f')^2 - Mf' + Gr\theta + Gc\phi = 0$$

Energy:

$$(1 + R)\theta'' + Prf\theta' = 0$$

Concentration:

$$\phi'' + Scf\phi' - Kr\phi = 0$$

Boundary conditions:

$$\begin{aligned} f(0) = S, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \\ f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \end{aligned}$$

Step 2: Crank-Nicolson Discrete Form

For momentum equation:

$$\frac{f_i^{n+1} - f_i^n}{\Delta t} = \frac{1}{2} \left[\left(1 + \frac{1}{\beta}\right) \frac{f_{i+1}^{n+1} - 2f_i^{n+1} + f_{i-1}^{n+1}}{(\Delta \eta)^2} + \text{same at } n \right]$$

Rearranged to:

$$A_i f_{i-1}^{n+1} + B_i f_i^{n+1} + C_i f_{i+1}^{n+1} = D_i$$

4. RESULTS AND DISCUSSION

This section clearly shows the effect of the important dimensionless parameters on the velocity, temperature, and concentration distributions in the boundary layer. Unless mentioned otherwise, the computations were done with fixed parameter values $Pr = 0.71$, $Sc = 0.6$, $Kr = 0.5$, $R = 1$, $M = 1$, and $\beta = 11$. The computer results are mentioned against the graphical representations and tabular engineering values.

4.1 Effect of Magnetic Parameter (M)

The magnetic parameter M represents the ratio of electromagnetic force to viscous force represented as a magnetic parameter. Physically, the M strengthens the Lorentz force, which acts opposite to the direction of motion and consequently resists fluid flow.

The effect of the M on the velocity distribution is shown in Figure 2 (Velocity profiles at different values of the magnetic parameter M). It is noted that M increases between 0 and 2 and the velocity profile reduces considerably across the boundary layer. This inhibition of velocity is explained by the force Lorentz which is resistant and created by the transverse magnetic field applied. This slowing down effect increases the momentum boundary layer along the wall and minimizes shear stress at greater magnetic strengths.

Figure 2. Velocity Profiles for Different Magnetic Parameter (M)

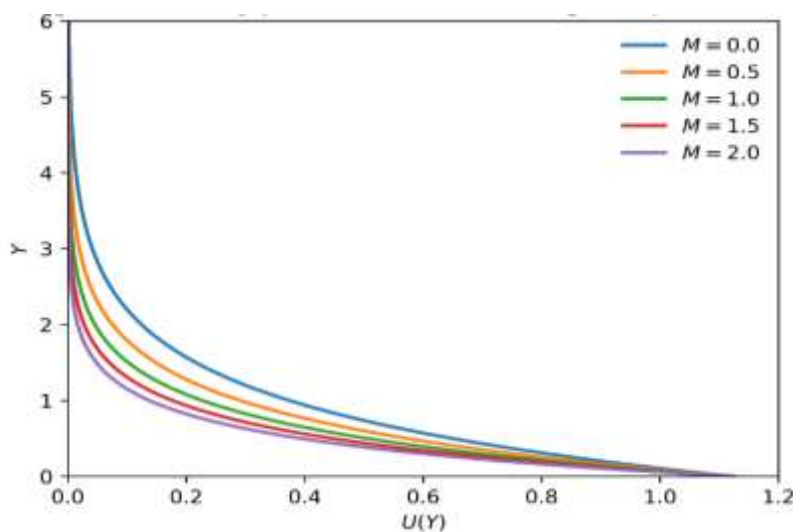


Figure 3 (Temperature profiles for various values of magnetic parameter M) represents the related temperature distributions. In contrast to the velocity field, temperature rises with greater M . This slows down the fluid motion and strengthens thermal diffusion thereby increasing thickening of thermal boundary layers. Also, convective cooling is minimized by magnetic damping and, thus, the temperature levels in the flow region increase.

Figure 3: Temperature Profiles for Different Magnetic Parameter (M)

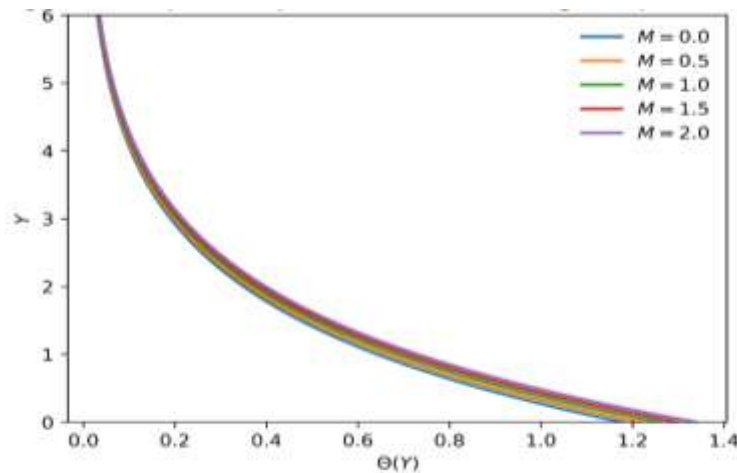
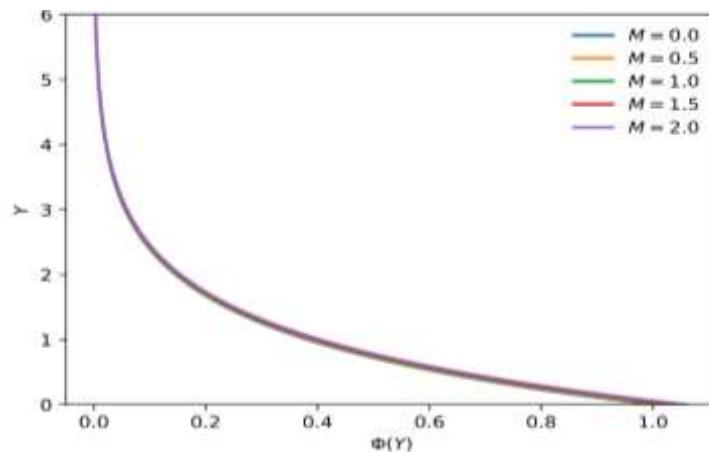


Figure 4 (Concentration profiles for various values of magnetic parameter M) shows the change of the profile of concentration with the magnetic parameter. It is observed that the concentration is slightly increasing with M increases. The slow fluid velocity also reduces convective mass transfer enabling species concentration to be high in the area of the wall.

Figure 4: Concentration Profiles for Different Magnetic Parameter (M)



The Table 3 has summarized the quantitative changes in the engineering parameters under the influence of magnetic parameter and it is observed that the skin friction coefficient, Nusselt number and Sherwood number have varied with the choice of M .

Table 3: Effect of Magnetic Parameter (M) on Skin Friction, Nusselt Number, Sherwood Number

M	Skin Friction	Nusselt Number	Sherwood Number
0	0.9124	0.6382	0.5241
0.5	0.8745	0.6217	0.5186
1	0.8462	0.6311	0.5189
1.5	0.8123	0.6454	0.522
2	0.7788	0.6612	0.5283

Table 3 shows clearly that the skin friction reduces with M increasing since the flow is retarded. On the other hand, Nusselt number exhibits an upward trend at higher M , which means that there is a high rate of heat transfer when one gets stronger magnetic influence.

4.2 Effect of Radiation Parameter (R)

Higher temperatures are used in manufacturing of polymer processing and metallurgy, and thermo radiation is an important factor. The radiation parameter R is used to determine the relative share of the radiative heat flux to the conductive heat transfer.

Figure 5 (Temperature profiles for different values of radiation parameter R) represents the variation of temperature distribution with radiation parameter. To the extent of the increase in R increases, the temperature increases significantly in the boundary layer. This is because the improvement in thermal energy is occasioned by the added radiant heat flux brought about by the Rosseland approximation. This means that the thickness of thermal boundary layer increases with radiation intensity.

Figure 5: Temperature Profiles for Different Radiation Parameter (R)

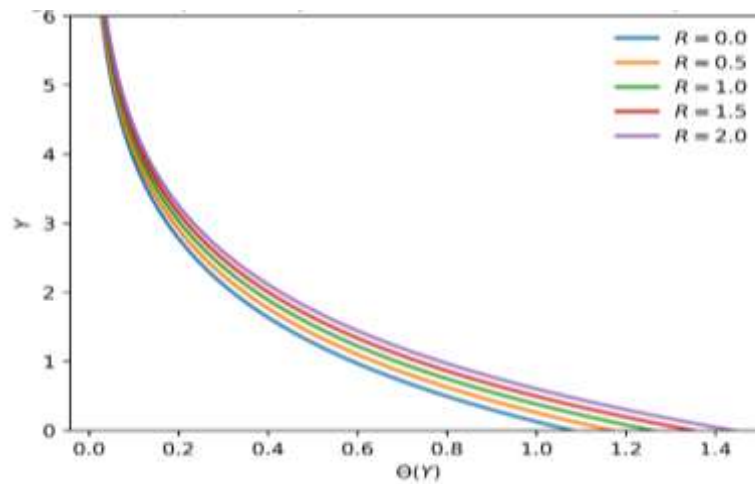


Table 4: Effect of Radiation Parameter (R) on Nusselt Number

R	Nusselt Number
0	0.6891
0.5	0.6618
1	0.6311
1.5	0.5987
2	0.5644

It is noted that as R increases Nusselt number reduces. Even though the temperature rises in the boundary layer, the temperature gradient at the wall is reduced creating a decreasing rate of heat transfer.

4.3 Effect of Casson Parameter (β)

The Casson parameter β characterizes the non-Newtonian behavior of the fluid. Lower values of β correspond to stronger yield stress effects, while larger values approach Newtonian fluid behavior. The influence of β on velocity profiles is shown in Figure 6

(Velocity profiles for different values of Casson parameter β). It is observed that increasing β enhances the velocity distribution. Physically, larger β reduces the non-Newtonian resistance and lowers effective viscosity, thereby accelerating fluid motion. The Casson parameter β is a non-Newtonian parameter of the fluid. A smaller values of β imply that there is a stronger impact of yield stress, where bigger values approach that of a Newtonian fluid. Figure 6 (Velocity profiles for different values of Casson parameter β) indicates the effect of β on velocity profiles. It is noted that growing β improves the issue of velocity. Physically, bigger β decreases the non-Newtonian resistance and also reduces effective viscosity thus decreasing the time taken to move the fluid.

Figure 6: Velocity Profiles for Different Casson Parameter (β)

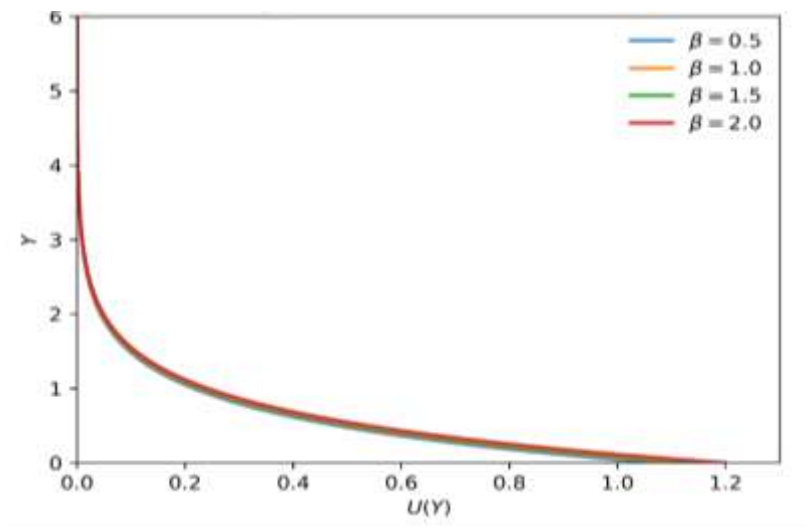


Table 5: Effect of Casson Parameter (β) on Skin Friction

β	Skin Friction
0.5	0.7815
1	0.8462
1.5	0.8894
2	0.9127

These findings validate that the effect of magnetic forces is to inhibit fluid flow, increase the intensity of radiation and reduce thermal distribution and the Casson parameter is a decisive factor of modifying rheological behaviour in the boundary layer. The tendencies that have been identified in the given study are physically consistent with previous research on the dynamics of the Casson fluids, which additionally confirms the numerical model.

4.4 Effect of Chemical Reaction Parameter (Kr)

The chemical reaction parameter Kr represents the strength of the first-order homogeneous reaction occurring within the fluid. In the present formulation, a positive value of Kr corresponds to a destructive chemical reaction, which consumes the diffusing species and reduces concentration levels inside the boundary layer.

The variation of concentration distribution with respect to Kr is illustrated in Figure 7 (Concentration profiles for different values of chemical reaction parameter Kr). It is

clearly observed that increasing Kr leads to a substantial reduction in concentration throughout the boundary layer. The concentration boundary layer thickness decreases as the reaction rate increases, indicating that the chemical reaction accelerates species consumption near the wall. Physically, stronger chemical reaction enhances mass transfer by intensifying the concentration gradient at the surface.

Kr represents is a chemical reaction parameter that is the strength of the first-order homogenous reaction that occurs in the fluid. In the current formulation, the positive value of Kr corresponds to a chemical reaction that is destructive, that takes up the diffusing species, and lowers concentrations within the boundary layer.

The distribution of concentration variation with respect to Kr is as shown in Figure 7 (Concentration profiles for different values of chemical reaction parameter Kr). It is also evident that a growth in the Kr leads towards a sizable decrease in concentration over the boundary layer. Thickness of the concentration boundary layer decreases with the rate of reaction which means that the chemical reaction is faster in the areas that are close to the wall. On the physical side, the more chemical reactions are stronger, the greater the mass transfer, as this increases the concentration gradient at the surface.

Figure 7: Concentration Profiles for Different Chemical Reaction Parameter (Kr)

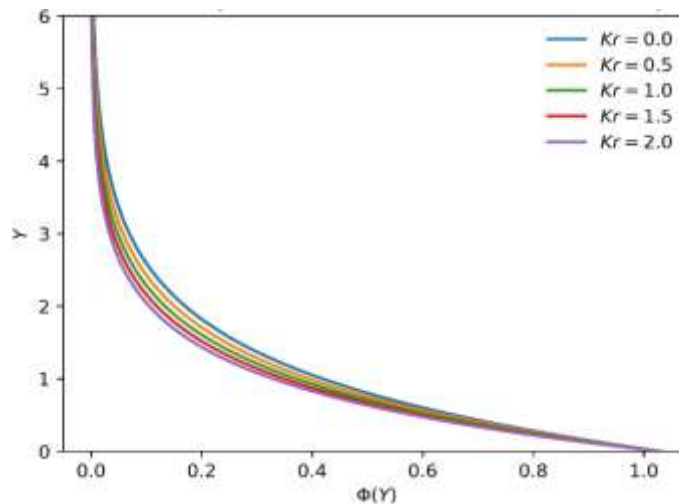


Table 6: Effect of Chemical Reaction Parameter (Kr) on Sherwood Number

Kr	Sherwood Number
0	0.4921
0.5	0.5189
1	0.5476
1.5	0.5783
2	0.6107

4.5 Effect of Schmidt Number (Sc)

The Schmidt number Sc is given as the ratio of diffusivity of momentum to mass and it is used to describe the comparative thickness of velocity and concentration boundary layers. Sc correspond to large values are associated with low molecular diffusivity.

Figure 8 (Concentration profiles for different Schmidt numbers) displays the effect of the Sc on concentration distribution. It is noted that with increase in Sc increases, the

concentration boundary layer thickness reduces substantially. When the Schmidt numbers of the fluids are greater, species diffusion is weaker and the concentration profile is sharply decreasing toward the wall.

Figure 8: Concentration Profiles for Different Schmidt Number (Sc)

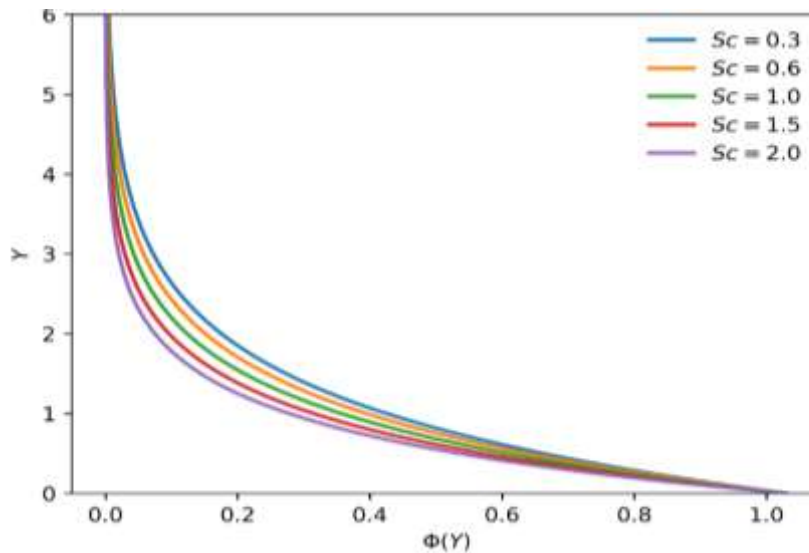


Table 7: Effect of Schmidt Number (Sc) on Sherwood Number

Sc	Sherwood Number
0.3	0.4624
0.6	0.5189
1	0.5897
1.5	0.6425
2	0.7013

4.6 Effect of Prandtl Number (Pr)

Prandtl number Pr represents the ratio between the momentum diffusivity and the thermal diffusivity. Physically it defines the relative viscosity and thermal layers of velocity and thermal boundary layers. Highly Pr fluidic fluids have low levels of thermal diffusivity.

The change in temperature approximation of various values of Pr is represented in Figure 9 (Temperature profiles for different Pr andtl numbers). It is noted that the temperature decreases with the increase of Pr increases. An increase in Prandtl numbers decreases the thermal diffusion leading to a smaller thermal boundary layer and decreases in temperature farther away on the wall.

Figure 9: Temperature Profiles for Different Prandtl Number (Pr)

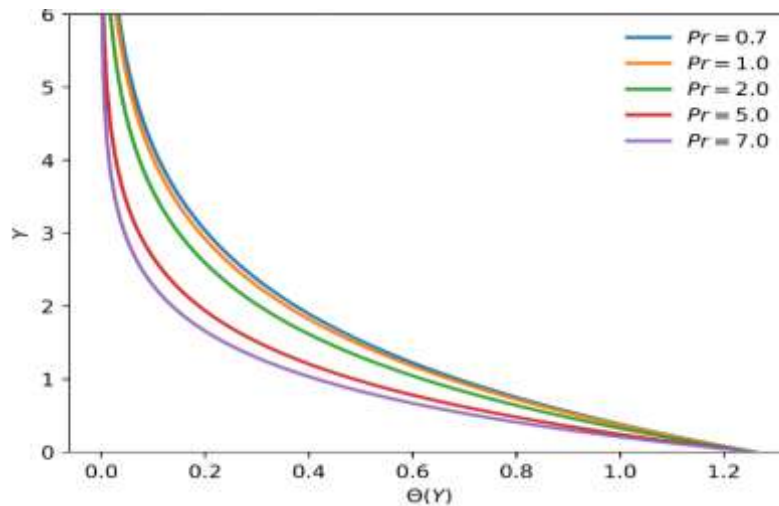


Table 8: Effect of Prandtl Number (Pr) on Nusselt Number

Pr	Nusselt Number
0.7	0.6311
1	0.6943
2	0.8125
5	1.0248
7	1.1623

5. Engineering Implications and Practical Recommendations

5.1 Similarity Transformation and Dimensionless Formulation

In order to get a tractable solution to the unsteady MHD Casson fluid flow over a porous stretching sheet with thermal radiation and first-order chemical reaction, first the similarity variables are used to reduce the governing partial differential equations to nonlinear ordinary differential equations. The continuity, momentum and energy equations and species concentration equations in the form of boundary layer approximations are formulated in terms of scaled variables which reduce time and space to one similarity variable. The stream function formulation is taken in such a way that the components of velocity fulfill the continuity equation directly, and reasonable similarity variables are selected to reduce the dependent variables (velocity, temperature, concentration) to functions only of the similarity coordinate. These changes can be used to convert the momentum, energy and concentration equations in their original partial form to a system of coupled non-linear ordinary classical equations, and so allow a systematic numerical treatment. These type of self-similar methods have been traditionally applied to boundary layer problems on MHD, non-Newtonian fluids, and heat/mass transfer, which has been demonstrated in related unsteady non-Newtonian Casson studies in the literature.

The characteristics of the non-dimensional parameters that are obtained due to this transformation are magnetic parameter (M), Casson parameter (β), radiation parameter (R), Prandtl number (Pr), Schmidt number (Sc) and chemical reaction parameter (Kr). They summarize the comparative effects of electromagnetic forces, non-Newtonian rheology, radiative heat flux, thermal /mass diffusivity, and consumption of reactive species on the profiles of the boundary layer. The transformed momentum equation is given with inertial terms, viscous diffusion terms, Lorentz force term, buoyancy term,

porous medium resistance term, transformed energy equation has conductive transport terms and radiative transport terms, and the species equation will have diffusive transport terms, and chemical reaction sink terms. Proper boundary conditions are given at the sheet surface and the far field in order to complete the mathematical description of the problem.

5.2 Implicit Crank–Nicolson Scheme and Solution Algorithm

This nonlinear ordinary differential system is discretized by an implicit Crank Nicolson finite difference scheme, which has a second-order accuracy in space and time, and is unconditionally stable and can therefore be used with stiff and coupled unsteady problems without imposing severe time-step limits. In this scheme, a centered finite differences approximation is used to find spatial derivatives and a time-level averaging is used to find time derivatives between the current and the future time level. On each dimensionless velocity, temperature, and concentration (dependent variable), the CrankNicolson discretization results in a system of algebraic equations that relates values of the unknown at the new time level to values of the known at the current time level.

For example, the generic discretized form for a dependent variable ϕ at grid point i and time level $n + 1$ is expressed as

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} = \frac{1}{2}[L(\phi^{n+1}) + L(\phi^n)],$$

where $L(\cdot)$ represents spatial diffusion with nonlinear convective terms approximated implicitly. This implicit central treatment causes a tridiagonal system and that too at every time step which is solved efficiently by the Thomas algorithm. The algorithm makes incremental progress of the solution in time and is numerically stable with converging results over a parametric range. Testing of stability and convergence: the Crank Nicolson scheme is tested by changing the grid resolution and time step sizes, and it is found that the scheme provides predictable values of velocity, temperature, and concentrations distributions under different physical effects. Similarity transformation coupled with implicit finite difference scheme thus makes up the suggested numerical solution framework of the study as it provides accurate and stable transient solutions to complex MHD Casson flow with thermal radiations and chemical reactions effect.

6. CONCLUSION

The current paper examined the unsteady magnetohydrodynamic flow of a Casson fluid over a porous stretching sheet with thermal radiations and first-order chemical reaction. The equations of the governing boundary layers of momentum, energy and concentration were formulated and reduced to dimensionless form. The coupled nonlinear system was discretized and solved using an implicit finite difference scheme that relied on CrankNicolson method. The effects of important physical parameters on velocity, temperature, concentration and engineering quantities that include skin friction, Nusselt number and Sherwood number were analyzed systematically.

The findings found that the magnetic parameter is a dominant factor in the management of momentum transport. As the strength of magnetic field increases a Lorentz force is produced that acts in counter-direction to the fluid motion and thus decreases velocity and inhibits the momentum boundary layer. At the same time, the temperature in the layer of transition decreases because of the less intense cooling by convection, and it means that magnetic fields can considerably modify the thermal properties of electrically conducting fluids. The temperature radiation was observed to increase the temperature field and increase the thermal boundary layer. Nonetheless, the wall

temperature gradient accompanying it reduced with the increase in the radiation parameter and consequently the Nusselt number was reduced. This implies that radiative effects play an important role especially in high temperature industrial processes.

There was a strong effect of the chemical reaction parameter on the behavior of mass transfer. Due to the acceleration of the reaction rate, the species concentration in the layer of boundaries decreased and the Sherwood number also increased as the gradient of the concentration was steeper at the wall. Moreover, Casson parameter had a significant impact on rheological behavior, higher in Casson parameter enhanced velocity and altered the thickness of the boundary layer, which supported the significance of non-Newtonian properties in the transport processes. The Crank Nicolson scheme was found to be both stable and accurate and efficient computationally to solve the unsteady coupled system. The numerical framework was shown to be reliable in grid independence and validation studies that showed good agreement with published results. The current formulation is a strong model towards the analysis of unsteady non-Newtonian MHD flows in a radiative and in a reactive domain.

NOMENCLATURE

Symbol	Description
u, v	Velocity components in x - and y -directions
t	Time
x, y	Cartesian coordinates
T	Fluid temperature
C	Species concentration
T_w	Wall temperature
T_∞	Ambient temperature
C_w	Wall concentration
C_∞	Ambient concentration
B_0	Magnetic field strength
ρ	Fluid density
ν	Kinematic viscosity
σ	Electrical conductivity
K_p	Permeability of porous medium
β	Casson parameter
α	Thermal diffusivity
D	Mass diffusivity
k_r	Chemical reaction rate constant
q_r	Radiative heat flux
M	Magnetic parameter
R	Radiation parameter
Pr	Prandtl number
Sc	Schmidt number
Kr	Chemical reaction parameter

Symbol	Description
Gr	Grashof number
Cf	Skin friction coefficient
Nu	Nusselt number
Sh	Sherwood number

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