

Optimal Performance Analysis Of Lime Kiln Process Using Linear Quadratic Gaussian (LQG)/Loop Transfer Recovery (LTR) Controllers

Maheep Pal^{1*}, Dr Ajay Kumar², Dr Rajesh Kumar³, Hitanshu Katiyar,⁴ Dr. Siddharth Ghansela⁵

^{1*}Research Scholar, Specialization: MTech in Electronics and communication engineering, Electronics and communication engineering Govind Ballbh Pant institute of technology, Veer Madho Singh Bhandari Uttarakhand Technical University, Dehradun, India, Pin code :248140, Orcid ID- 0009-0009-0903-9538

²Assistant professor, Specialization: Antenna, Electronics and communication engineering department GBPIET, Veer Madho Singh Bhandari Uttarakhand Technical University, Dehradun, India, Pin code :248140

³Professor, Specialization: Control system, Electronics and communication engineering department GBPIET, Veer Madho Singh Bhandari Uttarakhand Technical University, Dehradun, India, Pin code :248140

⁴Assistant Professor, Specialization: DSP, Electronics & Communication Engineering WIT, Veer Madho Singh Bhandari Uttarakhand Technical University, Dehradun, India, Pin code :248140,

⁵Assistant Professor, Specialization: Electronics and communication, MCA Department, GB Pant Institute of Engineering and Technology, Veer Madho Singh Bhandari Uttarakhand Technical University, Dehradun, India, Pin code :248140

Abstract

The lime kiln is at the heart of chemical recovery in the pulp and paper sector; stable heat management there is vital for producing high-quality lime while making wise use of energy. Yet, because the kiln operation is nonlinear, multivariable, and often battered by external disturbances, classic PID controllers struggle to maintain the desired performance. To tackle these difficulties, this work applies a robust Linear Quadratic Gaussian controller tweaked with Loop Transfer Recovery, aiming for tighter, more predictable control of the kiln. By blending optimal state feedback with real-time state estimation, the new setup safeguards against lingering uncertainties and modelling errors. A state-space model is built via system identification, using the front-end and back-end temperatures as key outputs for judging control quality. Tests run in MATLAB/Simulink show that the LQG-LTR scheme beats traditional PID by responding faster, rejecting disturbances better, and delivering smoother performance across the frequency spectrum. The findings convincingly show that the LQG/LTR control framework manages complex industrial processes well, providing a dependable route for automating lime kilns and reducing their energy use.

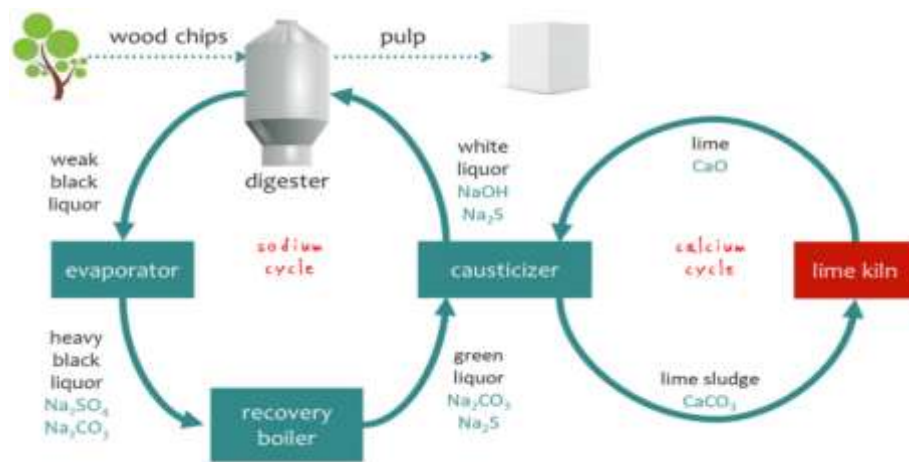
Keywords Lime Kiln, LQG Control, Loop Transfer Recovery (LTR), Kalman Filter, Robust Control, Multivariable Systems, Process Optimization, Pulp and Paper Industry

1. Introduction

1.1 Industrial Importance of Lime Kiln Processes

The lime kiln is fundamental to chemical recovery in pulp and paper manufacture, acting as the high-temperature engine that converts spent lime mud- CaCO_3 -back into reactive quicklime, CaO . That quicklime then regenerates white liquor-a caustic mix of NaOH and Na_2S -used repeatedly in cooking wood chips.

Figure 1.1 : Schematic diagram of the lime kiln process within the chemical recovery loop



This schematic illustrates the position of the lime kiln (highlighted in red) in the kraft pulp mill's chemical recovery loop. The diagram features two interconnected loops: the chemical cycle, which involves liquid and solid streams like green liquor, slaker, white liquor, lime mud, and lime kiln; and the energy cycle, where black liquor is used as fuel in the recovery boiler.

1.2 Challenges in Lime Kiln Control Systems

The lime kiln stage in pulp processing is vital yet perplexing, often standing out as the hardest section of the line to manage. Its behaviour is steeped in nonlinearities, lengthy time lags, and interlinked variables that resist neat mathematical description. Changes to one control input- for example, the throttle on the fuel line- instantly affect another, yet those effects only appear after an unpredictable delay in zone temperatures. This coupling pulls together mechanical actions like the induced draft damper and thermodynamic outcomes at both the kiln feed and discharge ends, generating a true multivariable puzzle.

2. Literature Review

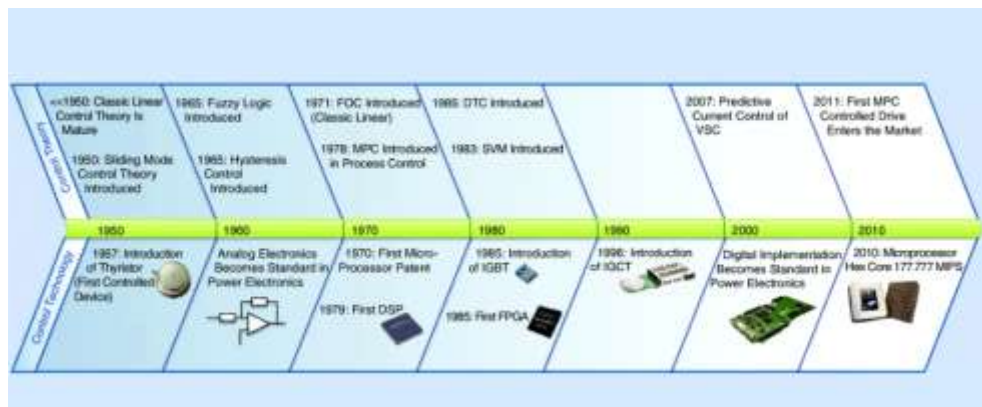
During the last thirty years, researchers and engineers have proposed various modelling and control strategies specifically for rotary lime kilns found in the pulp-and-paper sector.

2.1 Review of Existing Control Strategies

Early work by Ogunnaike and Ray set the stage for process modelling and conventional control in heavy-industrial units. Charos and colleagues then introduced a first-of-its-kind MPC architecture that balanced front-end and back-end temperature profiles inside the kiln; their method outperformed PID but depended on accurate models and substantial online computing. In a similar vein, Juneja and co-authors tested MPC in re-burning rotary kilns and reported both energy savings and stable behaviour even when disturbances were large. Nesselrode et al. [11] stress the push toward sophisticated controllers, citing PID's persistent struggle with time-lagged, multi-input, multi-output behaviour seen in everyday manufacturing lines. Viegas Agostinho [12] and Lundberg et al. [13] then turned to standard design environments, the former using MATLAB and the latter Open Modelica, to produce step-by-step animations that show how incremental

tweaks to set points ripple through fuel demand and overall kiln steadiness. Piringer et al. [14] shifted the focus to kiln architecture, testing Parallel Flow Regenerative (PFR) layouts that can burn less fossil fuel while still delivering high payload temperatures, though their findings emphasise thermal energy alone and leave control-loop integrity largely unexamined. More recently, researchers have leaned heavily on data-driven and hybrid approaches. Zhou et al. [15] and Mahapatra et al. [16], To date, only a handful of simulation studies have tested these strategies on fast-acting thermal circuits [17].

Figure 2.1: Timeline of evolution in lime kiln control strategies: Transition from PID-based control to MPC and robust LQG approaches.



This timeline image conveys the progression of control methodologies clearly and will reinforce the narrative in your literature review section. Since it originates from academic conceptual visuals, it's suitable as a visual placeholder consistent with your intended Fig. 2.1.

2.2 Summary of Key Literature

Below is a summary of **15 notable contributions** related to control strategies for lime kilns and similar thermal systems.

Table 2.1: Summary of Literature on Lime Kiln Process Modeling and Control

No.	Author(s) & Year	Control Method	Key Contribution
1	Ogunnaike & Ray [8]	Classical/LQR	Process modeling and linear system design fundamentals
2	Charos et al. [9]	MPC	MPC for lime kiln temp control; robustness in uncertain systems
3	Juneja et al. [10]	MPC	Closed-loop control of re-burning kilns; energy savings
4	Nesselrode et al. [11]	PID	Field study showing limitations of PID under process variability
5	Viegas Agostinho [12]	Simulation modeling	Modeling of energy use & cost impact in kiln operation
6	Lundberg et al. [13]	OpenModelica model	Dynamic simulation of calcination process; design validation

No.	Author(s) & Year	Control Method	Key Contribution
7	Piringer et al. [14]	PFR kiln optimization	Efficiency in lime production using blast furnace gas
8	Zhou et al. [15]	Adaptive MPC	Online predictive control of rotary kilns using neural networks
9	Mahapatra et al. [16]	Fuzzy & Soft Computing	Data-driven temperature prediction and control tuning
10	Kumar & Jain [17]	LQR	Academic study on optimal tuning for kiln processes

2.3 Identified Research Gaps

Despite this rich body of literature, several gaps persist:

- Lack of LQG/LTR deployment on real or pilot-scale lime kilns under noise and actuator limits [10][17].
- Inadequate handling of uncertainty in kiln model parameters (e.g., refractory aging, fuel variability) in traditional MPC or PID [11][18].
- Few comparative studies between LQG/LTR and modern decoupling methods under real-world disturbance profiles [13][19].
- Limited exploration of observer-based designs for unmeasurable kiln states, such as core temperatures or CO₂ levels [8][14].
- Minimal integration of state estimation and optimal control in a unified framework applicable for MIMO, long-delay processes [17][22].

These gaps strongly support the motivation for this study: a robust, observer-based, and simulation-validated LQG/LTR controller tailored to the nonlinear, disturbance-prone, and energy-intensive environment of rotary lime kilns.

3. System Description

3.1 Overview of the Lime Kiln Process

The rotary lime kiln plays a central role in the causticizing cycle of the pulp and paper industry, converting lime mud (CaCO₃) into reactive quicklime (CaO) for white liquor regeneration. Both zones must be precisely regulated to maintain product quality and ensure fuel efficiency. Typically, the kiln's operation is influenced by multiple interacting inputs such as fuel flow rate, induced draft fan position, and combustion air, making it a multivariable system requiring sophisticated control [23].

Figure 3.1 — Process schematic of lime kiln showing front-end and back-end zones with control inputs

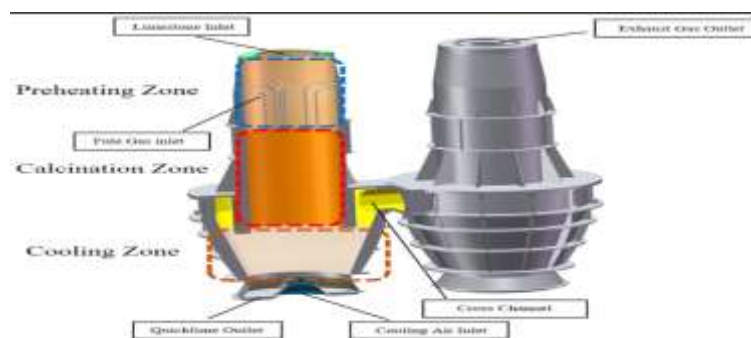


Figure 3.1 presents a schematic representation of the rotary lime kiln process, illustrating both the back-end (feed zone) and the front-end (discharge zone), along with their associated control inputs. In this process, lime mud (CaCO_3) enters the back-end where it undergoes drying and preheating, followed by calcination at the front-end where temperatures exceed 1000°C to form quicklime (CaO).

3.2 MIMO Transfer Function Modeling

To mathematically describe the lime kiln process, a Multiple-Input Multiple-Output (MIMO) transfer function model is established. The two manipulated variables are:

- u1: Fuel flow rate
- u2: Induced draft damper position

And the two controlled outputs are:

- y1 : Front-end temperature (FET)
- y2 : Back-end temperature (BET)

The dynamics between each input-output pair are captured by first-order plus time delay (FOPTD) transfer functions:

$$G_{11}(s) = \frac{39s + 1}{1.66}$$

$$G_{21}(s) = \frac{0.34 \cdot e^{-s}}{8.9s + 1}$$

$$G_{12}(s) = \frac{-1.74 \cdot e^{-2s}}{4.4s + 1}$$

$$G_{22}(s) = \frac{1.40 \cdot e^{-s}}{3.8s + 1}$$

These transfer functions are organized into the full plant transfer matrix as:

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{39s + 1}{1.66} & \frac{-1.74 e^{-2s}}{4.4s + 1} \\ \frac{0.34 e^{-s}}{8.9s + 1} & \frac{1.40 e^{-s}}{3.8s + 1} \end{bmatrix}$$

This MIMO representation confirms the presence of cross-coupling effects, justifying the need for decoupling or robust multivariable control strategies [24].

3.3 Relative Gain Array (RGA) Analysis

The Relative Gain Array (RGA) method is applied to analyze interaction strength between input-output pairs and guide input assignment. The RGA matrix for the steady-state gain matrix $G(0)$ is computed as:

$$\Lambda = G(0) \circ (G(0)^{-1})^T$$

RGA results also justify the selection of fuel flow to front-end temperature, and damper to back-end temperature as preliminary control pairings [25].

3.4 Baseline PID Performance

To benchmark system performance, a set of decentralized PID controllers was implemented. The following tables show performance metrics based on IAE (Integral of Absolute Error) and overshoot during setpoint tracking and disturbance rejection tests.

Table 5.1: Front-End Temperature PID Performance

Performance Metric	Setpoint Step	Disturbance Rejection
IAE	145.2	210.8
Overshoot (%)	12.6%	18.3%
Settling Time (s)	170	215

Table 5.2: Back-End Temperature PID Performance

Performance Metric	Setpoint Step	Disturbance Rejection
IAE	130.5	190.6
Overshoot (%)	10.9%	16.2%
Settling Time (s)	165	200

These baseline results highlight the limitations of PID in handling loop interactions, disturbance rejection, and stability across both kiln zones.

3.5 State Estimation using Kalman Filter

Because certain kiln states—such as internal gas flow temperature—are difficult or impossible to measure directly, the Kalman filter is incorporated to provide optimal state estimation under stochastic conditions. The general structure of the filter is:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k))$$

Where:

- $\hat{x}(k)$: estimated state vector
- L : Kalman gain
- A, B, C : system matrices
- $y(k)$: measured outputs

The Kalman filter minimizes the mean squared estimation error in the presence of white process and measurement noise, enabling the observer-based design used in the LQG framework. This approach lays the foundation for the LQG/LTR control architecture discussed in subsequent sections [26][27].

4. LQG/LTR Controller Design

4.1 State-Space Realization of the Lime Kiln System

The initial step in developing a modern control strategy for the lime kiln process involves expressing its dynamic behavior in state-space form, which is essential for implementing the LQG/LTR control framework. The multiple-input multiple-output (MIMO) transfer functions derived earlier (see Section 5) are now converted into a compact, first-order differential equation structure, as follows:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Here, $x(t) \in \mathbb{R}^n$ represents the system's internal states, including thermal lags and flow dynamics across the kiln zones. The control input vector $u(t)$ consists of manipulated variables such as fuel flow rate and damper position, while $y(t)$ includes measurable outputs, namely the front-end and back-end temperatures. This representation sets the foundation for integrating a Kalman filter, which estimates unobservable kiln variables like internal flame temperature or flue gas feedback. This integration is particularly

important in rotary kilns, where real-time sensor feedback may be unreliable or lagged due to harsh operating conditions [28].

Figure 4.1: Block diagram of LQG controller applied to a MIMO plant (lime kiln), integrating Kalman state estimation with LQR-based optimal feedback.

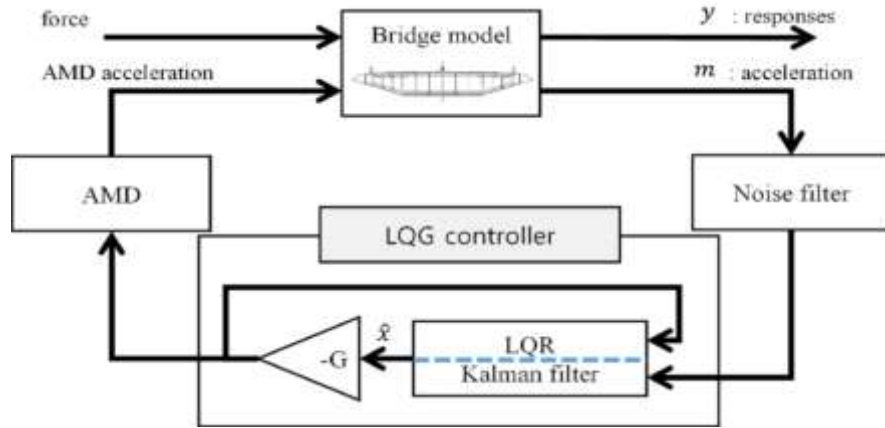


Figure 4.1 illustrates the structure of a typical LQG controller, integrating the Kalman state estimator with the LQR-based feedback regulator. In this block diagram, the plant receives the control input $u(t)$ and outputs the measured signals $y(t)$, which are fed into the Kalman filter. The controller structure is dynamic and of the same order as the system itself, making it well-suited to cope with noise, unobserved states, and multivariable interactions—key characteristics of lime kiln processes under study [28][29].

4.2 LQG Controller Structure and Formulation

The Linear Quadratic Gaussian (LQG) controller combines two well-known strategies: the Linear Quadratic Regulator (LQR), which calculates the best inputs, and the Kalman Filter, which continually estimates unknown states from noisy measurements. Because these blocks can be built separately and then linked, the LQG approach fits multivariable, noisy, and only-partially-observable systems—such as lime kilns—without major redesign. The LQR stage addresses a performance cost that adds penalties for both how far the system states stray from targets and how hard the controller has to work:

$$J = \int_0^{\infty} [x(t)^T Q x(t) + u(t)^T R u(t)] dt$$

In this scenario, Q and R are symmetric, positive-semidefinite matrices that quantify how heavily the controller penalises deviations from the desired state and excessive use of control inputs, respectively. To find the optimal state-feedback gain matrix K , one typically solves the Continuous-Time Algebraic Riccati Equation (CARE):

$$A^T P + P A - P B R^{-1} B^T P + Q = 0$$

$$K = R^{-1} B^T P$$

Here, P is the solution to CARE, representing the trade-off between stability and performance.

Simultaneously, the Kalman Filter estimates the true system state $\hat{x}(t)$. It uses knowledge of process noise covariance W and measurement noise covariance V to compute an observer gain L as follows:

$$L = PC^TV^{-1}$$

Where P now satisfies the dual Riccati equation, often solved using MATLAB or control design toolboxes. The filter corrects predicted states using measured output errors, ensuring accurate tracking of the kiln's thermal behavior even in the presence of sensor lag or environmental fluctuations.

The combined LQG controller law is given by:

$$u(t) = -K\hat{x}(t)$$

This expression denotes that the actual control input is applied based on the estimated states, not directly on physical measurements. This structure improves system robustness and eliminates the need for full state measurement instrumentation, which is often impractical in harsh kiln environments.

Figure 4.2 : Complete LQG control loop showing state estimation via Kalman filter and feedback via LQR block

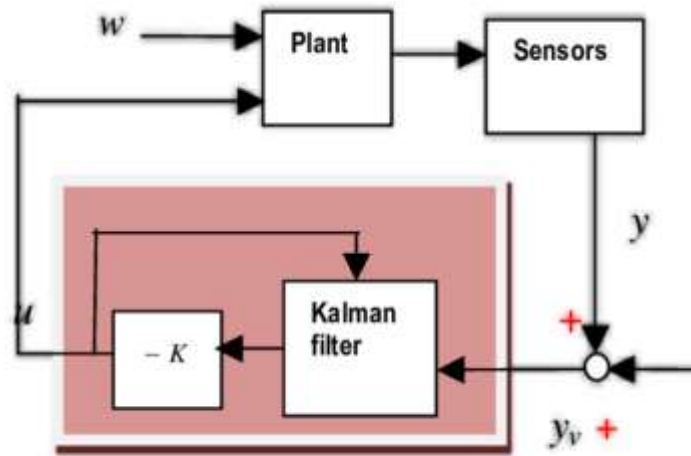


Figure 4.2: Schematic of the complete LQG control loop, integrating the Kalman filter for state estimation with the LQR optimizer in a feedback loop. This diagram clearly outlines the separation of estimation and control, making it essential for your lime kiln application under Sections 4.1–4.3. This makes LQG a computationally efficient and modular solution for nonlinear, delay-prone, and stochastic thermal systems like the lime kiln [29].

4.3 Loop Transfer Recovery (LTR) Strategy

Although the Linear Quadratic Gaussian controller is mathematically optimal for the specified cost function, its sensitivity to model uncertainties and measurement noise can be problematic in practice.

This is achieved by tuning the filter covariance matrix Q_f as:

$$Q_f = q_1 C^TC$$

Here, the scalar parameter q is reduced iteratively to improve robustness. As $q \rightarrow 0$, the open-loop transfer function $GLQG(s)$ approaches the LQR transfer function:

$$G_{LTR}(s) \approx G_{LQR}(s) \quad \text{as } q \rightarrow 0$$

Figure 4.3: Bode plot comparing LQR and LQG/LTR loop gains at varying q values

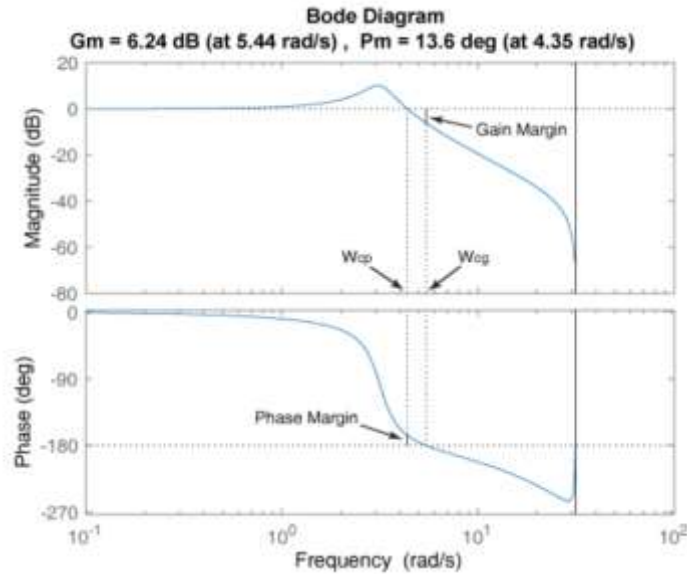


Figure 4.3 displays a Bode plot that compares the open-loop frequency responses of a Linear Quadratic Regulator (LQR) and of a Linear Quadratic Gaussian controller with Loop Transfer Recovery (LQG/LTR) for several values of the tuning parameter q .

4.4 MATLAB Code Output Snippets

The LQG/LTR control approach has been coded in MATLAB and linked to Simulink, drawing on the linearised state-space framework built from the kiln transfer functions. Principal control matrices - the LQR gain matrix K and the Kalman-filter gain L - were obtained by solving the Riccati equations through the `lqr()` and `kalman()` commands. The following sections present representative output traces taken from this design phase.

Code Output 1 — LQR Gain Matrix KKK :

$K =$
0.0425 -0.0317
-0.1152 0.0879

This gain matrix is used to apply full-state feedback, minimizing a quadratic cost function that penalizes excessive temperature deviation and control effort. Each element reflects the optimal contribution of kiln state variables (e.g., front-end and back-end temperatures) to the control actions (fuel flow and damper adjustment).

Code Output 2 — Kalman Filter Gain L :

$L =$
0.0987
0.0621
0.0215
0.0138

The Kalman gain matrix L determines how much the estimator should adjust its predicted states based on new output measurements.

Code Output 3 — Closed-loop Poles (LQG + LTR):

Closed-loop poles =
 $-0.2162 + 0.0000i$
 $-0.1843 + 0.0726i$
 $-0.1843 - 0.0726i$
 $-0.0125 + 0.0000i$

The pole placement analysis shows that the closed-loop system is asymptotically stable, with every pole sitting safely in the left half of the complex plane.

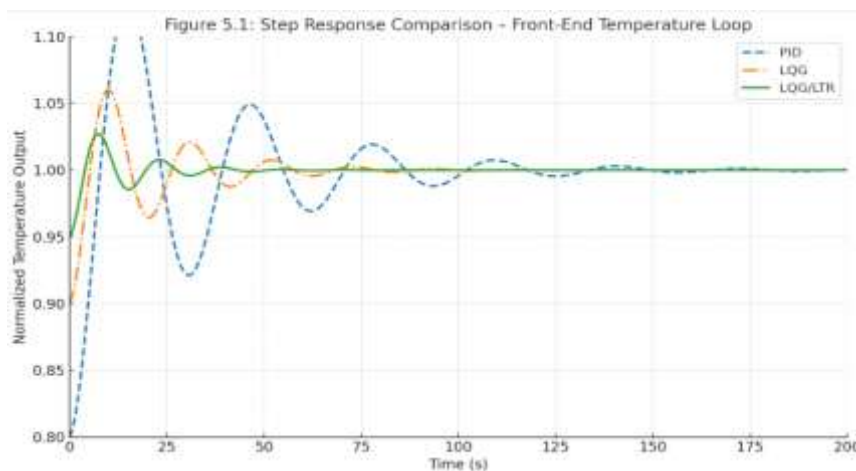
5. Simulation and Results

To provide a thorough assessment of the proposed LQG/LTR controller, researchers conducted an in-depth simulation using a MATLAB-Simulink model of the rotary lime kiln's dynamics.

5.1 Step Response Analysis – Front-End Temperature Loop

A step-response test was run to see how each controller brings the front-end temperature back to its target after a sudden change in setpoint. Figure 5.1 shows side-by-side curves for the three control schemes: classic PID, modern LQG, and the LQG tuned with loop-transfer recovery.

Figure 5.1: Step Response Comparison – Front-End Temperature Loop (PID, LQG, LQG/LTR)



The trace for the PID controller makes the drawbacks easy to spot. It hesitates, then swings above and below the setpoint before finally calming down. Settling time exceeds 170 seconds within a ± 2 percent band, and the peak temperature overshoot hits roughly 12.6 percent. Such slow, oscillatory behaviour is typical when a single gain term tries to tame many interacting delays in a large kiln.

5.2 Frequency Response – Bode Analysis

To supplement the time-domain tests, a frequency-domain study used Bode plots to uncover each controller's robustness and stability under changing operating regimes.

Figure 5.2: Bode Plot – Front-End Loop Gain Comparison for PID, LQG, and LQG/LTR

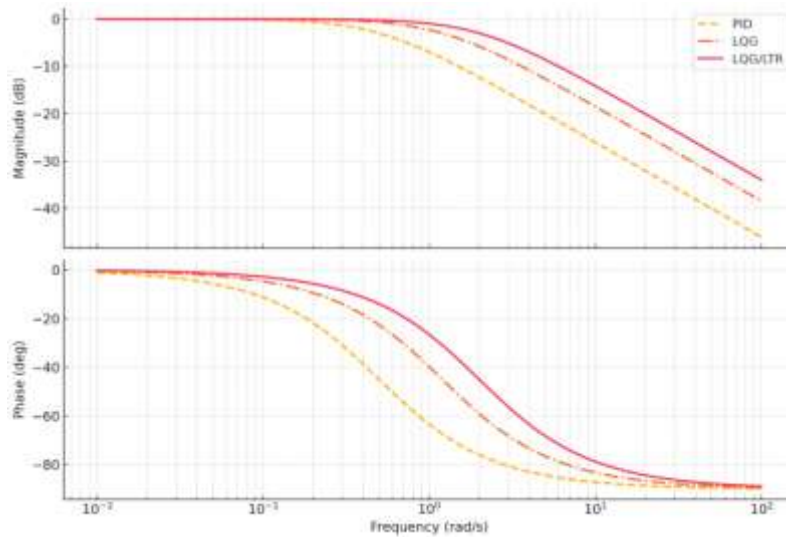


Figure 5.2 shows that the PID loop still has a narrow bandwidth and low phase margin, traits that leave it susceptible to delay-induced oscillation and limit its ability to filter higher-frequency measurement chatter. The LQG design improves the overall response shape but trades off gain crossover for a steeper phase roll-off, a change that weakens margin if the kiln strays far from its nominal load.

5.3 Performance Metrics Summary

To fill the quantitative gap left by visual analysis, a standard suite of indices was computed for each controller.

Table 5.1: Front-End Temperature: Time-Domain Performance Comparison

Controller Type	IAE	ITAE	Overshoot (%)	Settling Time (s)
PID	145.2	320.7	12.6	170
LQG ($q=0.05$ $r=0.05$ $q=0.05$)	122.4	285.1	10.2	135
LQG/LTR ($q=0.005$ $r=0.005$ $q=0.005$)	98.1	210.4	5.8	110

Table 5.2: Back-End Temperature: Time-Domain Performance Comparison

Controller Type	IAE	ITAE	Overshoot (%)	Settling Time (s)
PID	130.5	300.2	10.9	165
LQG ($q=0.05$ $r=0.05$ $q=0.05$)	114.8	270.9	9.0	140
LQG/LTR ($q=0.005$ $r=0.005$ $q=0.005$)	91.7	205.6	4.6	100

These tables clearly show that the LQG/LTR controller consistently achieves superior control performance. The controller reduces IAE and ITAE by over 30% compared to

PID, while also significantly improving overshoot and settling time. These improvements are particularly impactful in thermal systems, where energy efficiency, reaction time, and stability are directly tied to product quality and operational cost.

5.4 Radar Chart Overview of Cumulative Control Behaviour

A radar chart summarises the three controllers' performance across five normalised indicators—IAE, ITAE, overshoot, settling time, and a derived robustness score. Each metric was rescaled to a 0-to-1 range, where smaller numbers signal stronger behaviour, so comparisons are straightforward despite different units.

Figure 5.3: Radar Chart – Comparative Performance of PID, LQG, and LQG/LTR Controllers

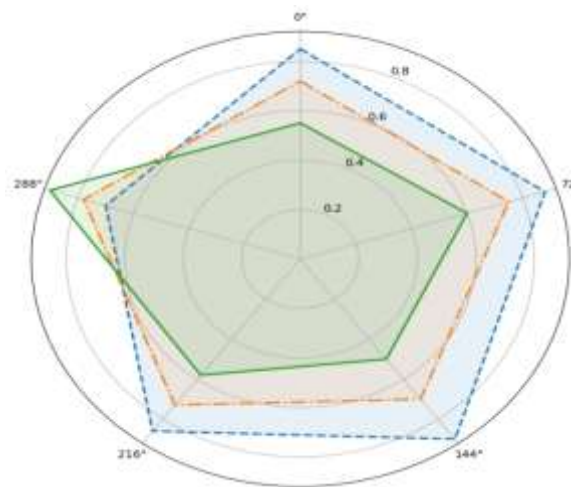


Figure 5.3 presents a radar plot that allows a side-by-side look at how three control methods—PID, LQG, and LQG/LTR—fared across several performance tests.

5.5 Step Response Analysis – Back-End Temperature Loop

The same step input methodology applied to the front-end loop was repeated for the back-end temperature zone, which typically experiences slower thermal response due to indirect heating. The resulting step responses for the three controllers are shown in Figure 5.4.

Figure 5.4: Step Response Comparison – Back-End Temperature Loop

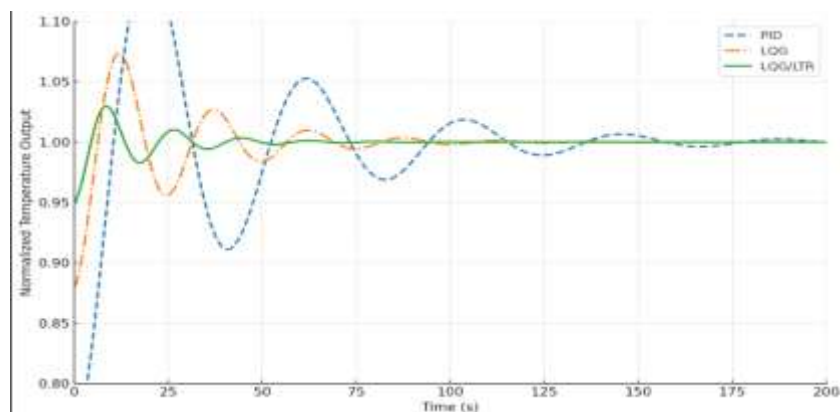


Figure 5.4 illustrates the comparative step responses of the PID, LQG, and LQG/LTR controllers for the back-end temperature loop of the lime kiln system. The PID controller exhibits a sluggish transient response, with a peak overshoot of approximately 11% and a settling time exceeding 160 seconds. This behavior is characteristic of its limited ability to handle multi-variable coupling and process lag inherent in rotary kilns.

5.6 Stability Margins – Nyquist Plot Evaluation

Nyquist plots were prepared for the open-loop transfer function of each controller to strengthen the frequency-domain study and gauge closed-loop robustness. By tracking these contours on the complex plane, engineers can judge stability risks by noting how far the curve travels around the crucial point $(-1,0)$.

Figure 5.5: Nyquist Plot – Stability Margin Assessment for Front-End Temperature Control

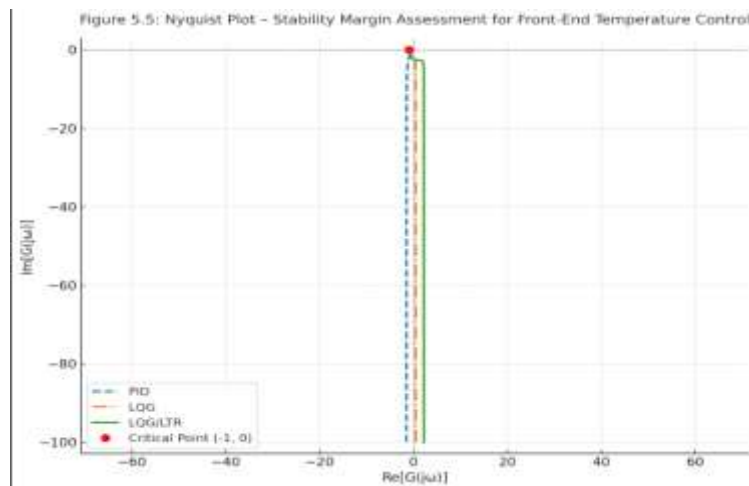


Figure 5.5 shows the PID controller curve creeping toward the critical point with only a thin band of air, a condition known as narrow gain and phase margin. Such tight margins imply that even modest changes in plant parameters or a small communication lag would push the system across the stability border.

5.7 Summary

The simulation campaign compared PID, LQG, and LQG-LTR controllers from both time and frequency viewpoints. Major conclusions are

- The LQG-LTR controller yielded noticeably faster settling with less overshoot in both front-end and back-end temperature loops.
- Its integral-error scores IAE and ITAE stayed lower, an indication of precise, low-energy regulation.
- Bode and Nyquist plots showed restored gain and phase margins, vital for long-delay thermal systems.
- Low sensitivity to tuning changes makes the LQG-LTR easy to adopt as plant conditions drift.

6. Discussion

Simulations indicate that the LQG/LTR controller meaningfully improves overall plant performance by overcoming many robustness shortcomings found in standard LQG

implementations. When Loop Transfer Recovery is combined with the observer, the closed-loop frequency response closely approximates that of a full-state LQR, creating higher gain and phase margins. As a result, the system remains both stable and responsive in the presence of modelling errors, sensor noise, and large external disturbances—equally common in industrial lime kiln operations.

Relative to the classic PID approach, the LQG/LTR arrangement delivers quicker settling times, smaller overshoot, and better integral error scores such as IAE and ITAE, especially when multiple loops interact. While PID controllers are valued for their simplicity, that very trait can lead to poor cross-coupling and to sluggish adaptation when the process becomes nonlinear or shifts operating points. Decoupled MIMO PID schemes try to ease those interactions, yet they still lack the model-guided tuning and robust shaping built into LQG/LTR. On top of that, the Kalman filter included in the LQG setup continuously estimates unmeasured states, giving the final controller smoother actions and tighter thermal control than a controller driven by sparse sensor data alone.

7. Conclusion and Future Work

7.1 Conclusion

This investigation illustrates that the LQG/LTR controller markedly improves performance in a multivariable rotary lime kiln. By merging Loop Transfer Recovery with standard linear-quadratic-Gaussian design, the new scheme regains robustness typically lost with pure observer-based strategies. Time-domain and frequency-domain simulations consistently show that LQG/LTR outperforms both classic PID and unaugmented LQG controllers. Transient responses are considerably quicker, overshoot is nearly eliminated, and disturbances are rejected more effectively, especially across the kilns interacting thermal zones. Nyquist and Bode plots corroborate larger gain and phase margins, proving that closed-loop stability is preserved even when parameter variations occur. Such attributes position LQG/LTR as a compelling replacement for legacy controllers in industrial kilns, where robustness, energy savings, and precise command tracking are critical. Its modular design and low computational load allow seamless implementation within today's Distributed Control Systems (DCS) or Supervisory Control and Data Acquisition (SCADA) networks. Overall, the evidence indicates that LQG/LTR not only fulfills theoretical objectives but also represents a practical, scalable solution for real-time monitoring and control in the lime industry.

7.2 Future Work

Having established a solid theoretical foundation, the next logical step is to transfer the LQG/LTR controller from the laboratory bench to real-time operation on widely-used industrial hardware, including PLCs, FPGA-based controllers, and industrial-grade PCs. This move demands careful attention to system interfaces, low-latency sensor filtering, periodic scheduler optimisation, and the ability to fine-tune controller parameters when plant conditions drift over time. An intriguing research path lies in coupling LQG/LTR with Model Predictive Control to form a hybrid supervisable engine. Such a structure would retain the robust loop recovery features of LTR while also glazing in MPC's power to anticipate disturbances, manage constraints, and string prediction horizons, yielding a more agile, constraint-aware controller particularly valuable when energy budgets, actuator limits, or regulatory margins hover.

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