

Analysis on the Frequency Response of Circular Membrane and Square Membrane Micromachined Ultrasonic Transducer

H. Lalnunfeli¹, Reshmi Maity², Ramesh Chandra Tiwari¹, Niladri Pratap Maity^{2,3}

¹Department of Physics, Mizoram University (A Central University), Aizawl – 796004, INDIA

²Department of Electronics and Communication Engineering, Mizoram University (A Central University), Aizawl - 796004, INDIA

³National Institute of Technical Teachers' Training and Research, Salt Lake City, Kolkata-700 106, INDIA

Corresponding Author: Niladri Pratap Maity

Abstract - The capacitance micromachined ultrasonic transducers (CMUT's) performance is significantly influenced by its shape. The objective of this research is to examine how different parameters of CMUT affects the resonant frequencies of two different shapes of CMUT. Our study investigates the differences between circular and square-shaped membranes, highlighting several key trends. The results demonstrate that thicker membranes and larger membrane radii lead to higher resonance frequencies. Additionally, a larger cavity radius results in lower resonant frequencies. The analysis also examines the effects of venting the membrane at the edge and center, revealing that edge venting decreases the device resonance frequency compared to a ventless membrane, providing valuable insights for CMUT design. The results of our study show that the circular-shaped membrane is more effective in achieving the desired CMUT frequency response, outperforming the square-shaped membrane in medical imaging system.

Keywords – CMUT, MEMS, COMSOL, FEM, Computational models

I. INTRODUCTION

Over the past two decades, researchers have developed various CMUT designs and computational models, as documented in numerous studies [1-5]. The application of modern micromachining techniques to electrostatic microphones marked a significant milestone. Compared to traditional piezoelectric transducers, CMUTs offer superior performance in terms of bandwidth, sensitivity and fabrication ease. As a Micro-Electro-Mechanical System (MEMS) based device, CMUTs leverage micromachining techniques to convert ultrasound waves into electrical signals and vice versa. This makes them an attractive option for both transmitting and receiving acoustic signals in the ultrasonic range. Researchers have developed various fabrication designs for CMUTs, including surface micromachining technique, bulk micromachining and wafer bonding. The surface micromachining technique, first employed by Khuri-Yakub et al. at Stanford University in 1994 [6], offers improved control over membrane dimensions, making it a valuable method for CMUT fabrication. In 2003, Khuri-Yakub et al. introduced a wafer bonding technique that enables precise control over gap height and utilizes single-crystal silicon membranes, allowing for predictable and controllable mechanical properties [7]. This technique includes anodic bonding, fusion bonding and adhesive bonding. By simplifying the fabrication process, wafer bonding reduces the number of steps and significantly improves turnaround time.

The performance of a CMUT in producing and detecting ultrasound signal hinges on its frequency response. In medical imaging, where precise visuals are crucial, a CMUT that can handle a broad range of frequencies is highly beneficial. This capability enables the device to generate images with superior resolution and detail, which is essential for accurate medical assessments [8-11].

A CMUT consists of a micromachined membrane suspended above a substrate, forming a small gap between the two [12-17], as illustrated in Fig. 1. When a voltage is applied across the membrane and substrate, a localized electric field is generated, causing controlled deflection of the membrane. This deflection modulates the capacitance between the membrane and substrate, allowing the device to precisely sense and actuate ultrasonic waves [17-22]. In this paper we have investigated of circular and square shape membrane CMUT device which performed using COMSOL Multiphysics for finite element method (FEM) simulation.

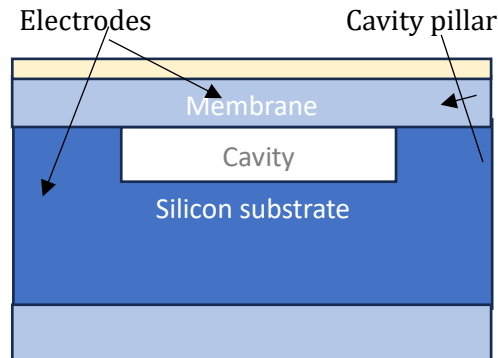
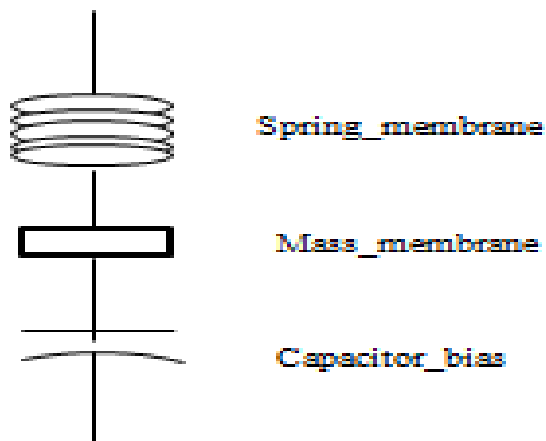


Fig. 1: Schematic diagram of CMUT

II. PROPOSED ANALYTICAL MODEL

The CMUT's circuit model is developed by assuming a linear relationship between the membrane's restoring force and displacement. Under the conditions of negligible electrical fringing force, perfect conductors and contacts, and vacuum operation, a lumped electro-mechanical model is employed to represent a single CMUT cell. This model consists of a spring, mass and parallel plate capacitor as shown in Fig. 2.

Fig. 2: Lumped electro-mechanical model



The force resulting from the membrane's mass F_m is equal to the sum of the capacitive force generated by the bias F_c and the force exerted by the membrane's spring-like behaviour F_k .

$$F_m = F_c + F_k \quad (1)$$

F_c is computed by differentiating the device's capacitance's potential energy U with respect to the membrane's displacement x_m .

$$F_c = -\left(\frac{dU}{dx_m}\right) \quad (2)$$

Thus, the force F_c due to the applied bias V , is expressed as

$$F_c = -\frac{d}{dx_m}\left(\frac{1}{2}C_d V^2\right) \quad (3)$$

The displacement of the membrane from its rest position is denoted by x_m and C_d represents the device capacitance. The spring, which models the membrane's restoring force can be assumed to be approximately proportional to the membrane's displacement. The force F_k exerted by the spring is determined by its bending stiffness and is given by,

$$F_k = K_m x_m \quad (4)$$

The spring constant K_m can be calculated by $\frac{16Y_m t_m^3}{a_m^2(1-\sigma_m^2)}$ (5)

Y_m represents the Young's Modulus of the membrane material, t_m is the membrane thickness, a_m is the membrane radius, and σ_m is the membrane material's Poisson's ratio. The natural frequency of vibration of the membrane with mass m is derived from (1) and is expressed as,

$$f_{\text{resonance}} = \left(\frac{1}{2\pi}\right) \sqrt{\left(\frac{K_m}{m}\right)} \quad (6)$$

The mass ' m ' is calculated using

$$m = 1.84\pi a_m^2 \rho_m t_m \quad (7)$$

Where ρ_m is the density of the membrane material.

III. FEM MODEL

Using COMSOL Multiphysics, the detailed structural parameters of our CMUT model are as shown in Table I. The meshing profiles for both the circular shape and square shape membranes are depicted in Figs. 3 and 4 respectively.

Table I. Structural parameters

Parameters	Values
Substrate radius	50 μm
Electrode radius (a_e)	50 μm
Membrane radius (a_m)	40 μm
Electrode thickness (t_e)	2 μm
Membrane thickness (t_m)	0.75 μm
Gap height (t_g)	1 μm
Poisson ratio (σ_m)	0.23

Young's modulus (Y_m)	250 GPa
Density (ρ_m)	3100 Kg/m ³

Fig. 3: Meshing profile of circular membrane CMUT

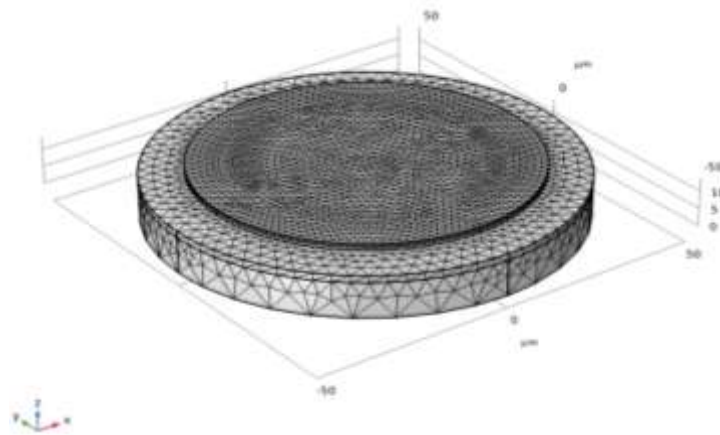
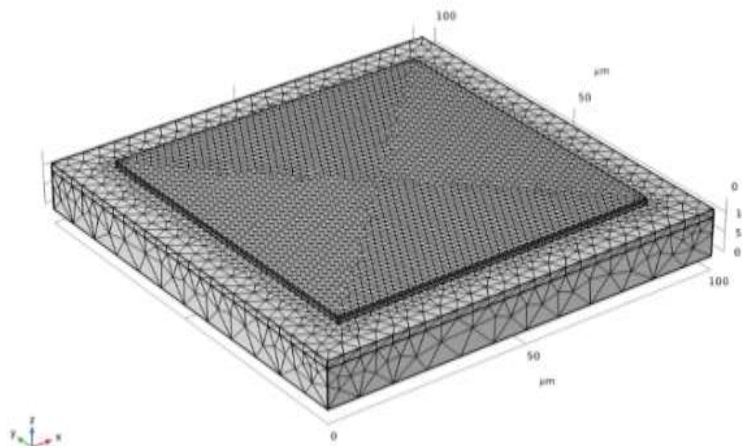


Fig. 4: Meshing profile of square membrane CMUT



IV. RESULTS AND DISCUSSIONS

Table II. Resonant frequency w.r.t Young's modulus of different materials

Membrane material	Young's modulus (GPa)	Circular membrane (MHz)	Square membrane (MHz)
Zinc Oxide (ZnO)	210	1.481	4.7595
Silicon Nitride (Si_3N_4)	250	2.121	6.794
Aluminium Oxide (Al_2O_3)	400	2.367	7.4803
Silicon Carbide (SiC)	748	3.918	12.091

The resonant frequency variations for circular and square shapes membrane with different Young's modulus values are graphically represented in Fig. 5, based on the data

in Table II. The results demonstrate that circular membranes yield lower resonance frequencies for equivalent Young's modulus values. Moreover, the resonant frequency increases in tandem with the Young's modulus, emphasizing the significant influence of membrane material properties on CMUT behaviour and performance.

Fig. 5: Resonant frequency w.r.t Young's modulus of different materials

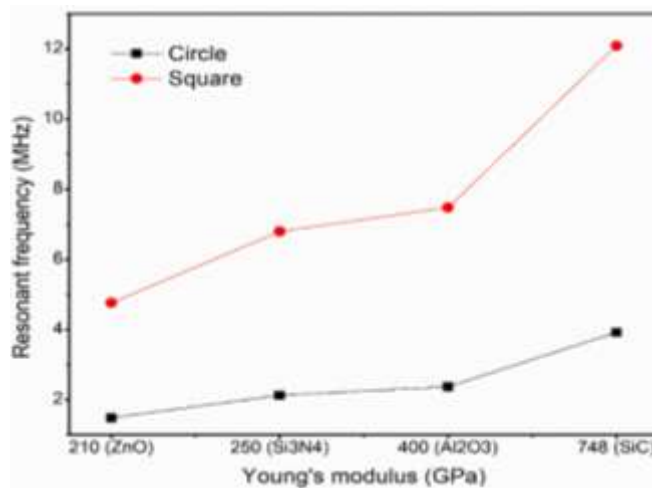
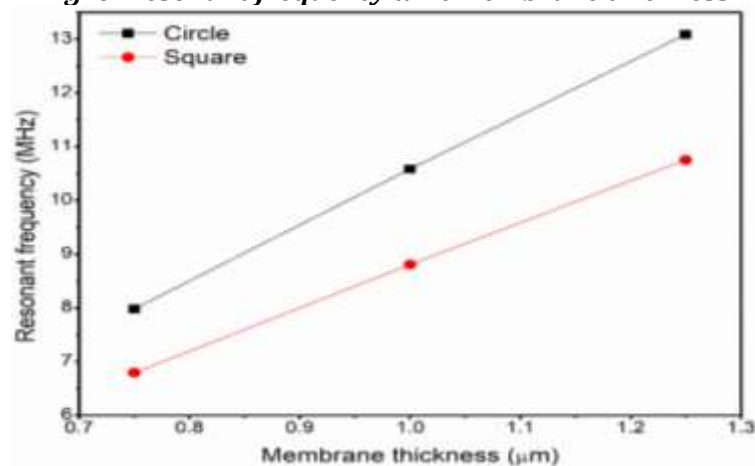


Table III. Resonant frequency w.r.t membrane thickness

Membrane thickness (μm)	Circular membrane (MHz)	Square membrane (MHz)
0.75	7.981	6.794
1	10.583	8.807
1.25	13.089	10.753

The graph in Fig. 6, corresponding to the data in Table III, displays the changes in resonant frequency for circular and square membranes with varying membrane thickness. A clear trend emerges, where thicker membranes result in substantially higher resonance frequencies. Furthermore, square membranes demonstrate lower resonant frequencies compared to their circular counterparts.

Fig. 6: Resonant frequency w.r.t membrane thickness



The harmonic displacement profiles for the square shape membrane are presented in Figs. 7 to 12.

Fig. 7: Displacement profile

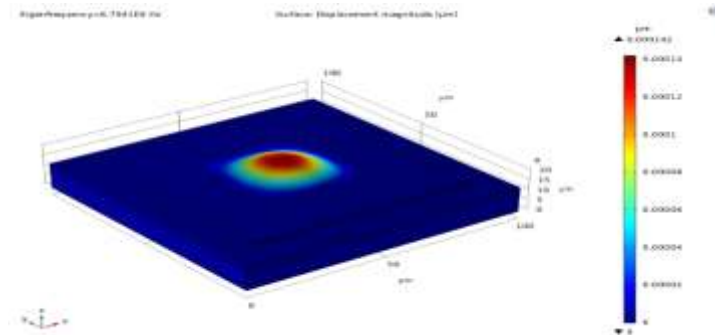


Fig. 8: 1st harmonic

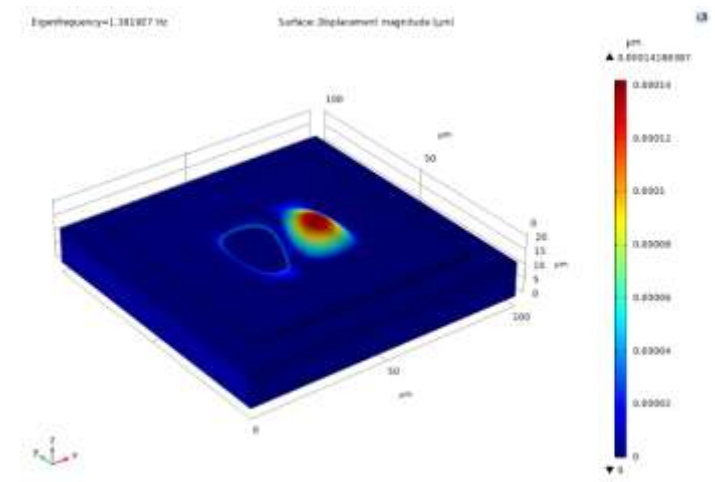


Fig. 9: 2nd harmonic

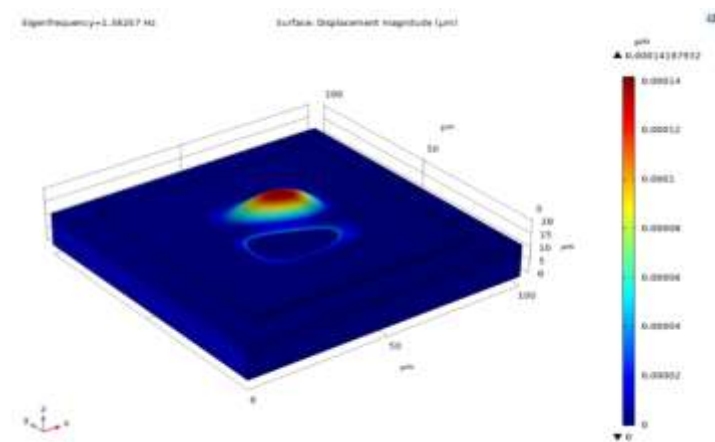


Fig. 10: 3rd harmonic

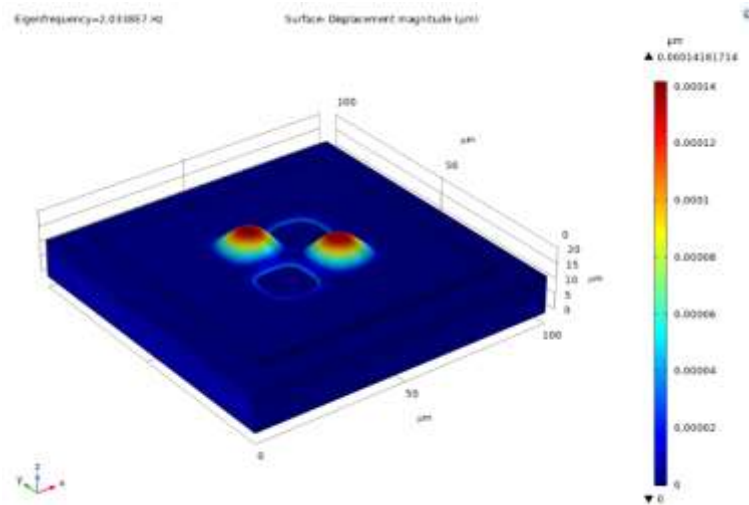


Fig. 11: 4th harmonic

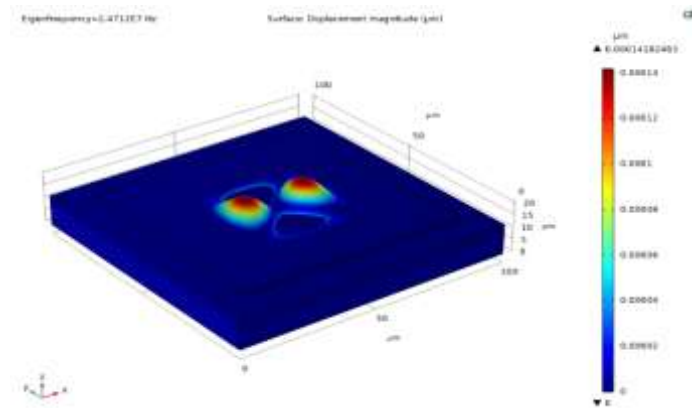


Fig. 12: 5th harmonic

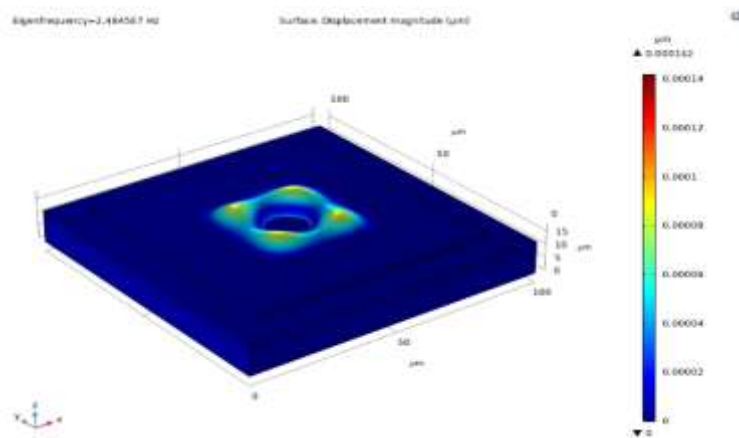
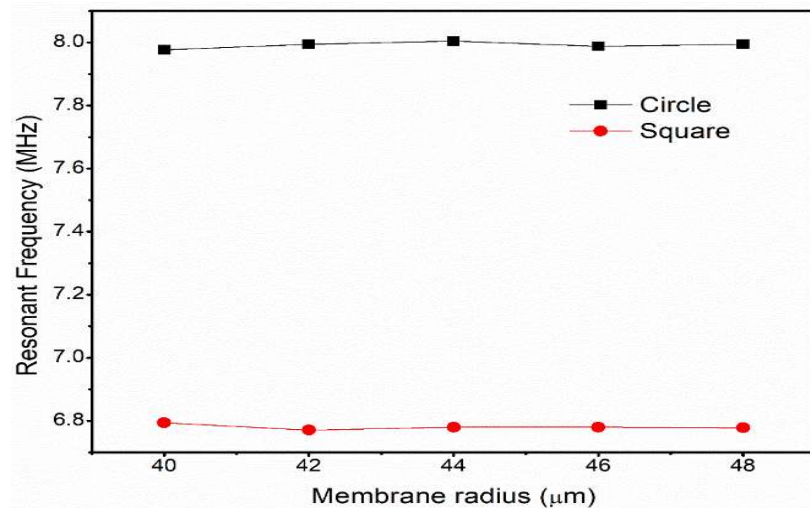


Table IV. Resonant frequency w.r.t membrane radius

Membrane radius (μm)	Circular membrane (MHz)	Square membrane (MHz)
40	7.977	6.794
42	7.994	6.771
44	8.005	6.780
46	7.988	6.78
48	7.994	6.778

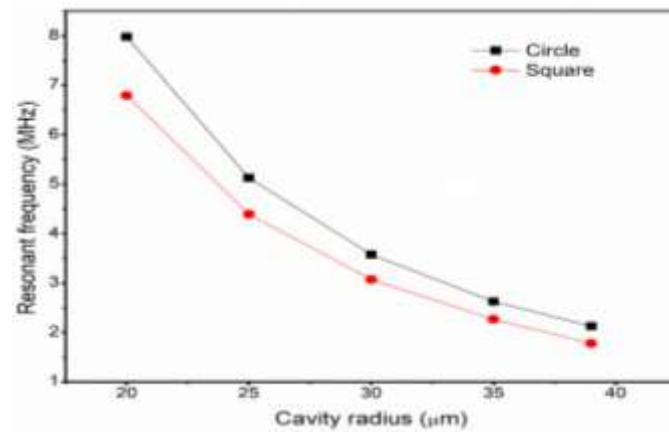
Fig. 13 plots the resonant frequency variations with respect to the membrane radius, as listed in Table IV. The plot reveals a direct relationship between membrane radius and device resonance frequency, with larger radii steadily yielding higher frequencies. Notably, the square-shaped membrane exhibits a lower device resonance frequency.

Fig. 13: Resonant frequency w.r.t membrane radius**Table V. Resonant frequency w.r.t cavity radius**

Cavity radius (μm)	Circular membrane (MHz)	Square membrane (MHz)
20	7.981	6.794
25	5.129	4.392
30	3.573	3.0702
35	2.625	2.264
39	2.127	1.776

Fig. 14 plots the resonant frequency variations with respect to cavity radius, as listed in Table V. The plot reveals an inverse relationship between cavity radius and resonant frequency, with larger radii yielding lower frequencies. Notably, the square membrane achieves the lowest resonant frequency, and the results underscore the importance of optimizing cavity radius.

Fig. 14: Resonant frequency w.r.t cavity radius



The displacement profiles of the circular membrane for various cavity radii (25 μm, 30 μm, 35 μm and 39 μm) are illustrated in Figs. 15-18, respectively. Similarly, the displacement profiles of the square-shaped membrane for the same cavity radii are shown in Figs. 19 to 22.

Fig. 15: Cavity radius of 25 μm

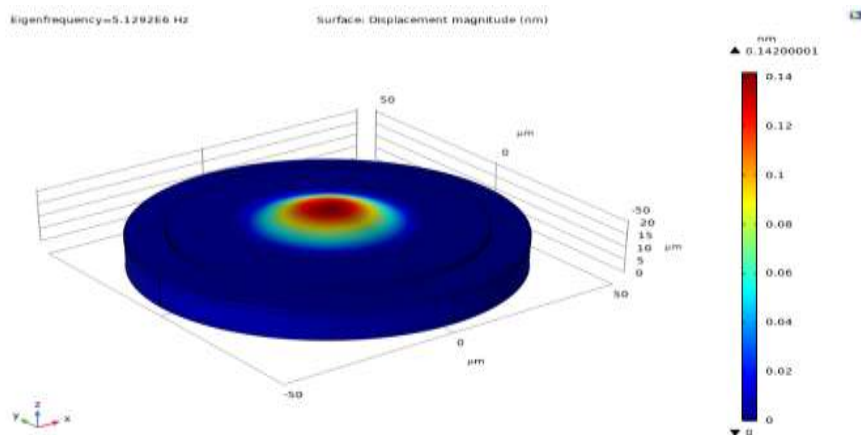


Fig. 16: Cavity radius of 30 μm

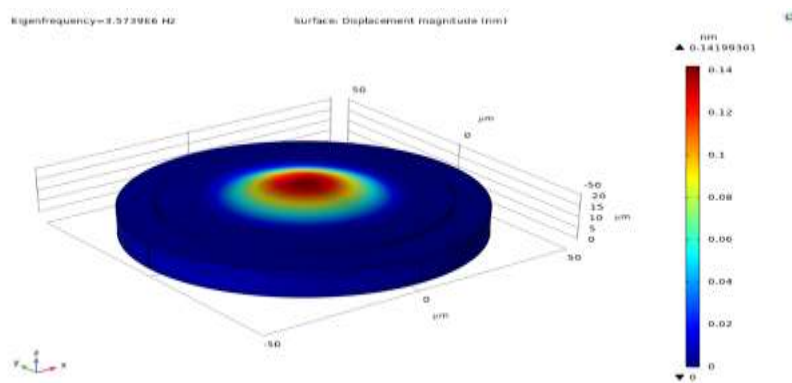


Fig. 17: Cavity radius of 35 μm

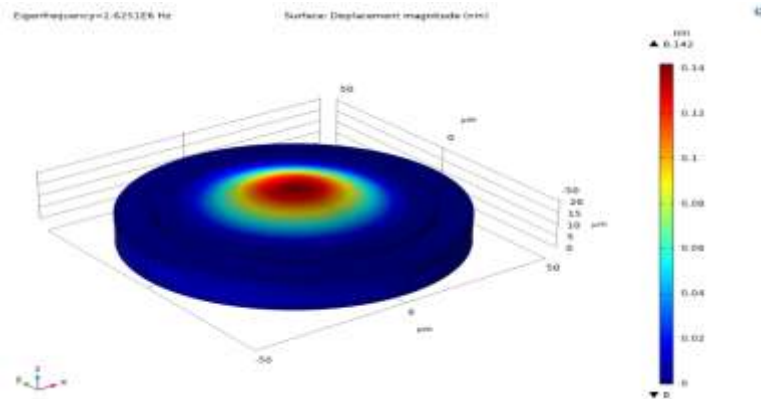


Fig. 18: Cavity radius of 39 μm

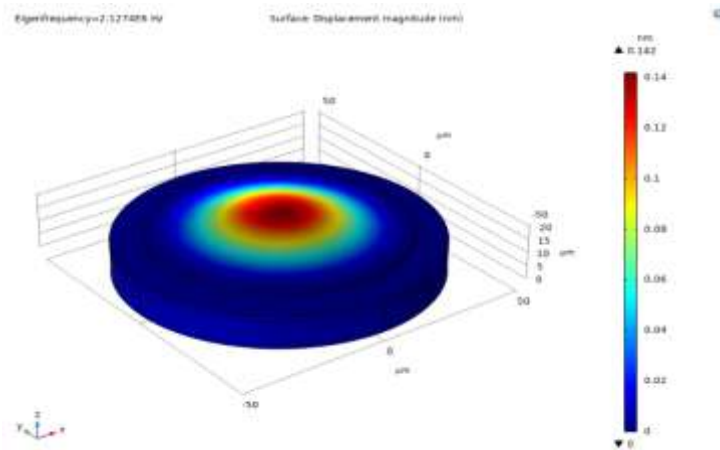


Fig. 19: Cavity radius of 25 μm

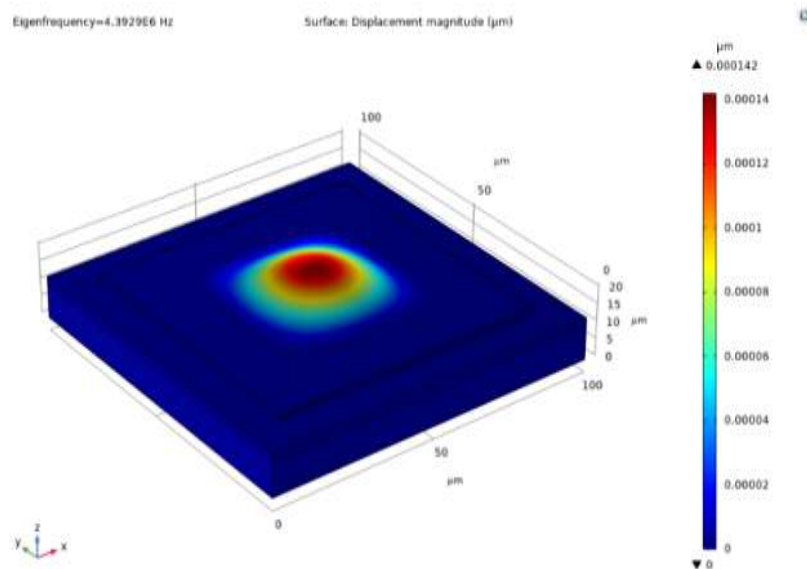


Fig. 20: Cavity radius of 30 μm

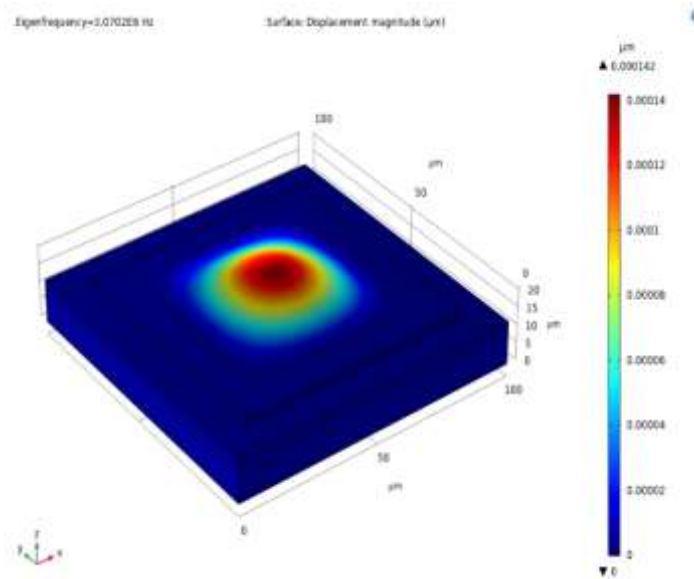


Fig. 21: Cavity radius of 35 μm

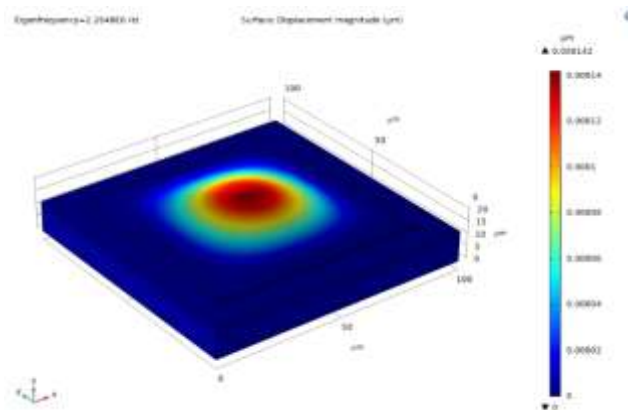


Fig. 22: Cavity radius of 39 μm

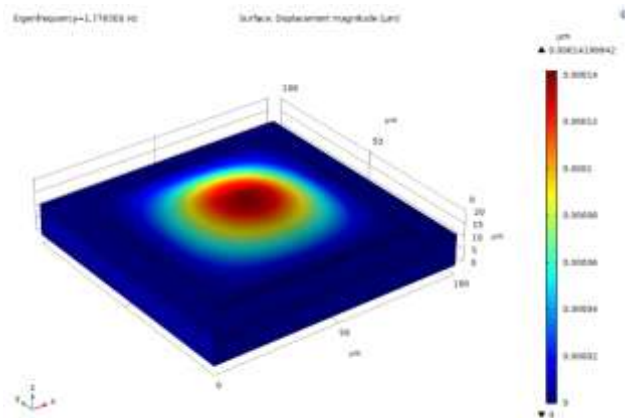
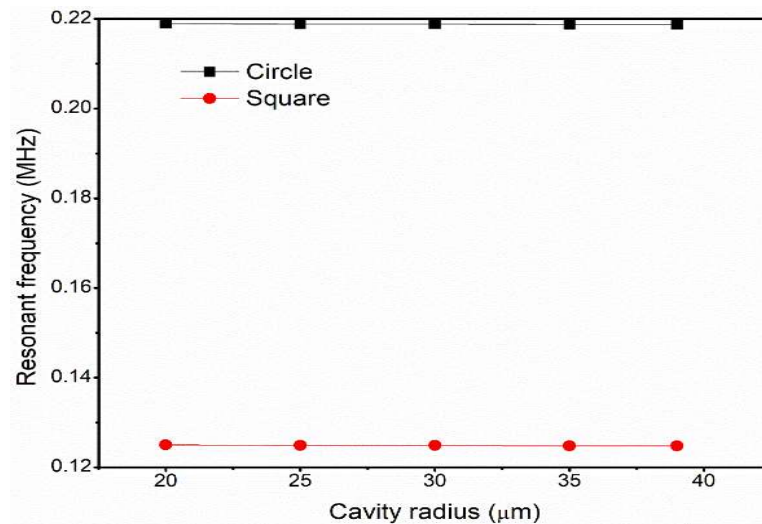


Table VI. Resonant frequency w.r.t air filled cavity radius

Cavity radius (μm)	Circular membrane (MHz)	Square membrane (MHz)
20	0.219	0.125
25	0.2188	0.1249
30	0.2188	0.1249
35	0.2187	0.1248
39	0.2187	0.1248

The resonant frequency variations with respect to air-filled cavity radius are graphically represented in Fig. 23, based on the data in Table VI. The results demonstrate that filling the cavity with air leads to a significant reduction in device resonance frequency. In contrast, increasing the air-filled cavity radius has a negligible effect on the device's resonance frequency, with only a marginal decrease evident from the graph.

Fig. 23: Resonant frequency w.r.t air filled cavity radius**Table VII. Resonant frequency w.r.t center vented radius**

Vented radius (μm)	Circular membrane (MHz)	Square membrane (MHz)
5	8.234	7.2645
6	8.683	7.7146
7	9.339	8.3444
8	10.223	9.1907
9	11.42	10.376
10	13.034	11.921

To provide further insight, we performed an analysis of the membrane's behaviour when vented at the edge and center for both circular and square shapes. The resonant frequency variations with respect to vented radius at the center of the membrane are graphically represented in Fig. 24, based on the data in Table VII. The results demonstrate a significant increase in resonant frequency with increasing vented radius. Conversely, Fig. 25,

representing the data in Table VIII, illustrates that venting at the edge of the membrane yields lower device resonance frequencies.

Fig. 24: Resonant frequency w.r.t center vented radius

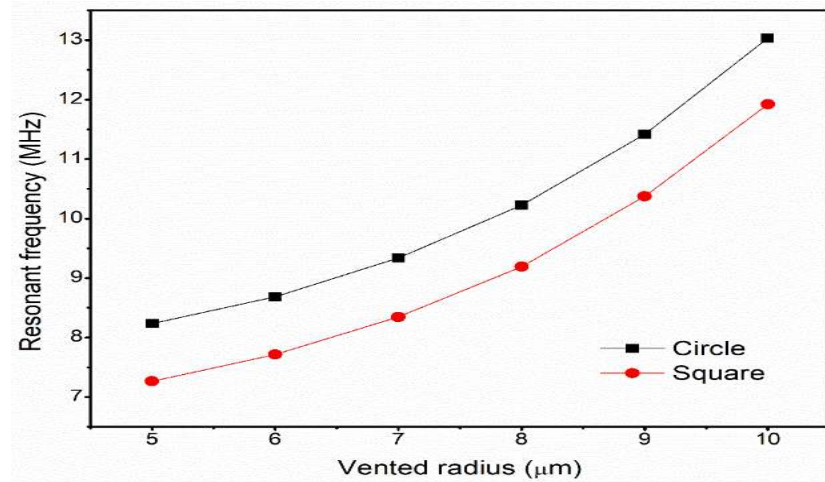
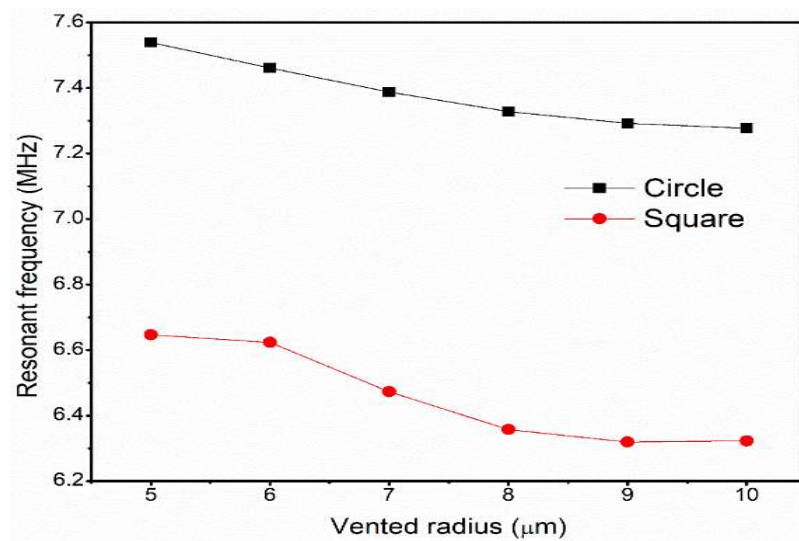


Table VIII: Resonant frequency w.r.t edge vented radius

Vented radius (μm)	Circular membrane (MHz)	Square membrane (MHz)
5	7.539	6.6468
6	7.461	6.6236
7	7.388	6.4732
8	7.328	6.3578
9	7.292	6.3196
10	7.277	6.3231

Fig. 25: Resonant frequency w.r.t edge vented circle



CONCLUSION

The investigation of circular and square shape membrane is performed using COMSOL Multiphysics. For the same parameters, higher resonance frequency is obtained from the circular shape membrane compared to the square membrane, proving that circular shape membrane is more effective in achieving the desired CMUT frequency response, which results in higher resolution in imaging system. The analysis also explores the impact of venting the membrane at different locations, specifically at the edge and center. Notably, the findings indicate that edge venting results in a decrease in the device's resonance frequency when compared to a membrane without venting. Thus, these observations offer valuable insights for the design of CMUTs, highlighting the significance of venting configuration in determining the device's performance characteristics.

REFERENCES

1. Haller M. I. and Khuri-Yakub B. T., "A surface micromachined electrostatic ultrasonic air transducer", IEEE Transactions on Ultrasonic, Ferroelectrics, and Frequency control, 1996, 43(1), 1-6.
2. Hohlfeld K., Michaelis A., Gebhardt S., "Piezoelectric transducers on the basis of free-formed PZT components," IEEE Proceedings of Joint International Symposium on Applications of Ferroelectric and Piezoresponse Force microscopy, DOI: 10.1109/ISAF.2013.6748675, 2013.
3. Y. Zhao et al., "Coupling Effects of Crosstalk and Parasitic Loss on Capacitive Micromachined Ultrasonic Transducers," IEEE Sensors Journal, vol. 22, no. 4, pp. 3281-3297, Feb. 2022
4. C. Hmingthansanga, M. Reshmi, D. Shankar, N. P. Maity, Investigation and analysis of MEMS based capacitive sensor's characteristics with the effect of electrode pattern, Microsystem Technologies, vol. 31, pp. 987-997, 2025
5. Khuri-Yakub B. T. and Oralkan O., "Capacitive Micromachined Ultrasonic Transducers for medical imaging and therapy", Journal of Micromechanics and Microengineering; 21 (5): DOI: 10.1088/0960-1317/21/5/054004, 2011.
6. Haller M.I. and Khuri-Yakub B.T., "A surface micromachined electrostatic ultrasonic air transducer". IEEE Ultrasonic Symposium, France, 1994, pp 1241-1244.
7. Huang Y., Ergun A.S., Haeggstrom E, Badi MH, Khuri-Yakub BT, " Fabricating capacitive micromachined ultrasonic transducers with wafer-bonding technology". Journal of Microelectromechanical Systems, 2003, Vol. 12, No. 2.
8. Maity R., Maity N.P., Thapa R.K., Baishya S., "Analysis of frequency response behaviour of capacitive micromachined ultrasonic transducers," Journal of Computational and Theoretical Nanoscience, 2015, Vol. 12, 3492-3494.
9. C. Hmingthansanga, M. Reshmi, N. Bijit, K, D, Shakar, N. P. Maity, Performance analysis of FEM simulated different shaped membranes based capacitive MEMS sensor, Microsystem Technologies, vol. 31, pp 461-473, 2025
10. Huang X., Wang H., Yu L., "Investigation on design theory and performance analysis of vacuum capacitive micromachined ultrasonic transducer," Micromachines 2021, 12(9), 1127.
11. Thacker M.B., Emadi A., Buchanan D.A., "Design, development, and characterization of a low frequency CMUT-based anemometer", 2021, DOI 10.1109/ACCESS.2021.3112107.
12. H. Lalnunfeli, M, Reshmi, R. C. Tiwari, D. Shankar, N. P. Maity, Modeling and Investigation of Immersion based Capacitive Micromachined Ultrasonic Transducer, Microsystem Technologies, 30, 1315-1323, 2024

13. Ray A., Chandel V., Taiwade K., Bamne J., Bhargav A., Haque F.Z., "Design and analysis of CMUT device using COMSOL for radio frequency applications," *materials Today: Proceedings* 2022, Vol. 65, 2602-2605.
14. Mills D. M., and Smith S. W., "Multi-layered PZT/polymer composites to increase signal-to-noise ratio and resolution for medical ultrasound transducers", *IEEE Transactions on Ultrasonic, Ferroelectrics, and Frequency control* 46(4), 961-971, 1999.
15. Avik Ghosh Dastidar, Reshmi Maity, R. C. Tiwari, Shankar Dutta, N. P. Maity, An Analytical and FEM Simulation-based Study of the Dependence of Capacitance Profile on Structural Parameters of CMUT with and without Vent, *Microsystem Technologies*, 30, 1239-1248, 2024.
16. Avik Ghosh Dastidar, Moumita Pal, R. C. Tiwari, R. Maity, N. P. Maity, An efficient electrostatic actuation model for MEMS-based ultrasonic transducers with fringing effect, *Microsystem Technologies*, 29, 583-597, 2024
17. Jin X., Ladabaum I. and Khuri-Yakub B. T., The microfabrication of capacitive ultrasonic transducers", *IEEE/ ASME J. Microelectromechanical Systems*, 1998, 7(3).
18. Avik Ghosh Dastidar, Reshmi Maity, R. C. Tiwari, D. Vidojevic, T. Kevkic, V. Nikolic, S. Das, and N. P. Maity, "Squeeze Film Effect in Surface Micromachined Nano Ultrasonic Sensor for Different Diaphragm Displacement Profiles', *Sensors*, vol. 23, no. 10, p. 113712.
19. M. Pal, C. Lalengkima, R. Maity, S. Baishya, N. P. Maity, "Effects of fringing capacitances and electrode's finiteness in improved SiC membrane based micromachined ultrasonic transducers," *Microsystem Technologies*, vol. 27, pp. 3679-3691, 2021.
20. Bijit Kumar Nath, Reshmi Maity, Niladri Pratap Maity, "Enhanced capacitance modeling of a circular microelectromechanical system based transducer for sensitivity improvement", *Microsystem Technologies*, vol. 31, No. 7, pp. 1863-1874
21. O. J. Adelegan, Z. Coutant, X. Wu, F. Yamaner, and O. Oralkan, "Design and Fabrication of Wideband Air-Coupled Capacitive Micromachined Ultrasonic Transducers With Varying Width Annular-Ring and Spiral Cell Structures," *IEEE Trans. on Ultrasonics Ferroelectrics and Frequency Control*, vol. 68, no. 8, pp. 2749–2759, Apr. 2021.
22. H. Shahid, D. D. Berhanuddin, R. Latif, P. C. Ooi, and T. Batool, "Simulation-Based Approach to Detecting Pulmonary Embolism Using Capacitive Micromachined Ultrasonic Transducers," 2024 IEEE International Conference on Semiconductor Electronics (ICSE), pp. 152–154, Aug. 2024.