PO- Ternary Γ- Radicals In PO- Ternary Γ- Semirings

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Abstract
This paper presents an investigation of Γ-radicals in the setting of PO-ternary Γ-semirings. We define the notion of Γ-radicals appropriate to partially ordered ternary operations and analyze their key algebraic characteristics. The study examines connections between radicals, ideals, and congruence relations, emphasizing the role of the order structure. Several foundational results concerning existence, characterization, and homomorphic behavior are obtained. The work broadens radical theory to ordered ternary semiring systems and establishes directions for further exploration.

Key Words: prime PO Γ- radical**,** completely prime PO Γ- radical, Prime, Semiprime, Completely semiprime.

INTRODUCTION

In 1934, Vandiver [6] introduced the concept of semiring. In point of fact, semiring is a overview term for ring. Lister [4] dubbed the algebraic system a ternary ring in 1971 for the additive subgroups of rings that are closed under the triple ring product. K. S. Dutta [1], Kar initiated the first ideal and the first radical of ternary structures. The same investigators begin irreducible ideals as well as semiprime ideals of [2]ternary semirings. Additionally, S. Kar has an idea of quasi- ideals also bi-ideals in ternary semirings. Likewise, in [5], we collect the necessary information on partially ordered radicals in partially ordered ternary semirings and present Bashir.S and M.Shabir prime bi-ideals in ternary semigroups. Throughout this paper, ternary Γ –semiring- TΓS, PO-ternary Γ-semiring- POTΓS,
commutative PO-ternary Γ-semiring- CPOTΓS, Prime PO-ternary Γ-ideal- PPOTΓI, completely prime PO-ternary Γ-ideal- CPPOTΓI, semiprime PO-ternary Γ-ideal- SPOTΓI, Completely semiprime PO-ternary Γ-ideal- CSPOTΓI, globally idempotent PO-ternary Γ-semiring- GIPOTΓS, commutative semigroup-CSg

1.PRELIMINARIES

The necessary preliminaries are discussed in this section.

Definition 1.1:

If T is an additive CSg that gratifies subsequent states, it is named as a TΓS. A TΓS consists of a set T($\neq \phi$), a binary action known as addition, moreover a ternary product indicated by [].

i) [[d γ eγ f] γ g γh]=[dγ [e γ f γ g] γ h]=[d γ e γ [f γ g γ h]],
ii) [(d+e) γ f γ g]=[d γ e γ f]+[e γ f γ g],
iii) [d γ (e+f) γg]=[d e γ g]+[d γ f γ g],
iv) [d γ eγ (f+g)]=[dγ e γ f]+[d γ e γ g] ∀ d; e; f; g; h ∈T, for γ in Γ.

Note1.2*:* For the ease we write $x_1x_2x_3$ instead of [x_1 γx_2 γx_3]
Note1.3: Permit T be a TΓS. If D, E & F are 3 subsets of T, we shall signify the set DΓEΓF ={∑d γ e γ f : d∈D, e∈E,f∈F}, for γ in Γ.
Note1.4*:* Let T be a TΓS. If C ⊆ T, D ⊆ T signify the set C+D={c+d : c ∈ C, d ∈ D} &

2C= {c +c: c ∈ C}.
Note1.5*:* A semiring can be reduced to a TΓS

Definition1.6: A TΓS T is called a POTΓS afford T is poset ∋ b ≥ a subsequently
(1) c+a ≤ c+b, a+c ≤ b +c
(2) a γc γd ≤b γc γd, c γa γd ≤ c γb γd & c γd γa ≤ c γd γb ∀ a, b, c, d ∈T, for γ in Γ.
Throughout T will indicate as POTΓS unless otherwise stated.

Note1.7: Sometimes we inscribe e ≥ f for e ≤ f. ie., "≥" is the dual of "≤".
Theorem1.8: Let T be a POTΓS also H⊆T,I⊆T & J⊆T. Followed by (i) H ⊆ (H] (ii) ((H]] = (H] (iii) (H](I](J] ⊆ (HIJ] (iv) H ⊆ I ⇒ H ⊆ (I] & (v)H ⊆ I ⇒ (H] ⊆ (I],
(vi)(H∩I] = (H]∩(I] (vii) (H∪I] = (H]∪(I].

Definition1.9**:** A subset C($\neq \phi$) of a POTΓS T is a POTΓI of T if C is additive subsemigroup of T, CΓTΓT⊆C, TΓTΓC ⊆ C, TΓCΓT ⊆ C & (C] ⊆ C.

Definition1.10*:* A POTΓI 'D' of a POTΓS 'T' is called a CPPOTΓI of T if u, v, w ∈ T and uγvγw ∈ D, for γ in Γ, ⇒ u ∈ D or v ∈ D or w ∈ D,

Definition1.11: Let T be a TΓS. A($\neq \phi$)⊆T is described a PO-c-system of T iff or ∀ c, d, e ∈ A ∃ an element f ∈ A ∋ f ≤ c γd γe, for γ in Γ

Definition1.12*:* A subset C($\neq \phi$) of a POTΓS T is called a PO-m**-**system offered for any g, h, i ∈ C ∃ j∈ C & x, y ∈ T ∋ j ≤g γx γh γy γi, for γ in Γ

Definition1.13*:* Allow T be a POTΓS. A($\neq \phi$)$\subseteq$ T is called a PO-d-system of T iff or every d ∈ A ∃ an component c ∈ A ∋ c ≤ d^n ∋ odd $n \in N$.

Definition1.14*:* A subset A($\neq \phi$) of a POTΓS T is called a PO-n-system afford for any c ∈ A ∃ d ∈ A & v,w ∈ T ∋ d ≤ cγvγcγwγc, for γ in Γ.

Theorem1.15: A POTΓI 'B' of a POTΓS T is completely prime iff T\B is whichever PO-c-system or Φ.
Theorem1.16**:** Each CPPOTΓI of a POTΓS T is a PPOTΓI.
Theorem1.17: Let T be a CPOTΓS. A POTΓI 'P' of T is a PPOTΓI iff P is a completely prime.
Theorem1.18: If T is a GIPOTΓS afterward each maximal POTΓI of T is a PPOTΓI.

Definition1.19: A POTΓI A of a POTΓS is identified a PPOTΓI of T provided Q, R, S are POTΓI's of T & QRS ⊆ A ⇒ Q ⊆ A or R ⊆ A or S ⊆ A.

Definition1.20*:* A POTΓI 'A' of a POTΓS g T is described a CSPOTΓI afford y ∈ T, y^n ∈ A ; $n \in N$ n >1 ⇒ y ∈ A.

Definition1.21*:* A POTΓI A of a POTΓS T is named SPOTΓI offerd Y is a POTΓI of T & Y^n ⊆ A, ∃ $n \in N$ ⇒ Y ⊆ A.

Theorem1.22: Each CPPOTΓI of a POTΓS T is a CSPOTΓI of T.
Theorem1.23**:** Let T be a CPOTΓS. A POTΓI A of T is completely semiprime iff it is semiprime.

Theorem1.24: Each PO-m-system is a PO-n-system in a POTΓS T.
Theorem1.25: A POTΓI Q of a POTΓS T is a SPOTΓI iff T\Q is Φ or PO-n- system.

2.PRIME PO-RADICAL AND COMPLETELY PRIME PO-RADICAL
Notation2.1: If A is a POTΓI of a POTΓS T, followed by we associate the next 4 kinds of collections.
A_1= All CPPOTΓI's of T whose intersection holding A.
A_2= {y ∈ T: $y^n \in A, n \in N$ }
A_3= All PPOTΓI's whose intersection of T holding A.
A_4={y ∈ T: $<y>^n \subseteq A,\ n \in N$ }
***Theorem2.2*:** If A is a POTΓS T , afterward A ⊆ A_4⊆ A_3⊆ A_2⊆ A_1
Proof :
i) A ⊆ A_4 : Let v ∈ A. Afterward $< v > \subseteq A$ & v ∈ A_4 , Consequently A ⊆ A_4
ii) $A_4 \subseteq A_3$: Let v ∈ A_4 . $\Rightarrow < v >^n \subseteq A$; odd $n \in N$.
Allow P be any PI of T holding A.
Next $< v >^n \subseteq A \Rightarrow < v >^n \subseteq P$; odd $n \in N$,While P is prime $< v > \subseteq P$ & v ∈ P.
And is right for every prime PO-ternary bi deals having A, $v \in A_3$, $\therefore A_3 \supseteq A_4$
iii) $A_2 \supseteq A_3$: Let v ∈ A_3, Believe if possible v ∉ A_2.
Next $v^n \notin A\ \forall$ odd $n \in N$, Consider $Q = \cup v^n\ \forall$ odd $n \in N$, v ∈ T.
Let h, i, j ∈ Q, Next $h = (v)^r, i = (v)^s, j = (v)^t$; $r, s, t \in N$.
$\therefore hij = (v)^r(v)^s(v)^t = v^{r+s+t} \in Q$ also Q is a PO-c-system.
Via Th.1.15, P = T\Q is a CPPOTΓI & v ∉ P.
Via Th.1.16, P is a PPOTΓI of T & v ∉ P.
Therefore $v \notin A_3$. It is a disagreement. $\therefore v \in A_2$, $A_3 \subseteq A_2$
iv) $A_2 \subseteq A_1$: Let v ∈ A_2 , Now v ∈ $A_2 \Rightarrow v^n \in A$; odd $n \in N$.
Allow P be any completely biprime POTΓI of T holding A
Next $v^n \in A \subseteq P \Rightarrow v^n \in P \Rightarrow$ v ∈ P. Therefore v ∈ A_1
$\therefore A_2 \subseteq A_1$ Consequently A ⊆ A_4⊆ A_3⊆ A_2⊆ A_1.

Corollary2.3: If A is a POTΓI of a POTΓS T, subsequently(A] ⊆ (A_4] ⊆ (A_3] ⊆ (A_2] ⊆ (A_1].
Proof: It is simple to confirm the proof via Th.1.8 & Th.2.2.

Theorem2.4: If A is a POTΓI of a CPOTΓS T, next A_1=A_2=A_3=A_4
Proof*:* Via Th.2.2, A ⊆ A_4 ⊆ A_3 ⊆ A_2 ⊆ A_1.
Via Th.1.17, in a CPOTΓS T, an ideal A is a PPOTΓI if A is CPPOTΓI.
SoA_1 = A_3. By Th.1.23, in a CPOTΓS 'T' a POTΓI A is semiprime iff A is CSPOTΓI.
$\therefore A_4 = A_2$ & $A_1 = A_2 = A_3 = A_4$.
Corollary2.5: If A is a POTΓI of a CPOTΓS T, next (A_1]=(A_2]=(A_3]=(A_4].
Proof: It is simple to confirm the proof via Th. 1.8 & Th.2.4.

Note2.6*:* In an arbitrary POTΓS (A_1]≠ (A_2]≠(A_3]≠(A_4].

Example 2.7: Let T be the free TSG produced via k,l,m . It is obvious A = (T Γ k^3 Γ T] is a POTΓI. As $k^5 \in$ (T Γ k^3 Γ T], k ∈ (A]. Evidently $(k \gamma l \gamma m)^n \notin$ (T Γ l3 Γ T] ∀ odd $n \in N$ and hence $k \gamma l \gamma m \notin$ (A]. Accordingly (A_2] is not a POTΓI of T.
Therefore (A_1] ≠ (A_2] & (A_2] ≠ (A_3]
Currently we initiate prime PO-radical in a POTΓS is also complete prime PO-radical.
Definition2.8: If A is a POTΓI of a POTΓS, afterward all PPOTΓI's whose intersection of T

having A is named as prime PO-radical of A moreover it is indicated by$\sqrt{A}$ or radA.

Definition2.9: If A is a POTΓI of a POTΓS, subsequently all CPPOTΓI's whose intersection of T enclosing A is named as complete PO-radical moreover indicated it via c.rad A.
Note2.10*:* If A is a POTΓI of a POTΓS T, next radA = A_3,c.radA = A_1 & radA ⊆ c.radA**.**

Note2.11*:* If (C] is a POTΓI of a POTΓS T, next rad(C] = (C_3], c.rad(C] = (C_1**]** & rad(C] ⊆ c.rad(C]**.**

Corollary 2.12: If d∈ $\sqrt{(C]}$, ∃ a odd $n \in Z^+$ ∋ $d^n \in (C]$.
Proof: Via corollary 2.3, (C_3] ⊆ (C_2] & d∈ $\sqrt{(C]} = (C_3] \subseteq (C_2]$.
∴ $d^n \in (C]$; odd $n \in Z^+$.

Corollary2.13: If F is a POTΓI of a POTΓS T, afterwards rad (F] = c. rad(F].
Proof: Via corollary2.5, c.rad (F]=rad (F].

Corollary2.14: If nA is a POTΓI of a POTΓS T subsequently c. rad(A] is a completely semiprime POTΓI.

Proof: Via Th.1.22, c. rad(A] is a CSPPOTΓI.
Theorem2.15: If H, G, F are any three POTΓI**'s** of a POTΓS T, followed via
(i) F ⊆ G ⇒ $\sqrt{F} \subseteq \sqrt{G}$
(ii) If F⋂G⋂H ≠ ∅ afterward $\sqrt{FGH} = \sqrt{F \cap G \cap H} = \sqrt{F} \cap \sqrt{G} \cap \sqrt{H}$
(iii) $\sqrt{\sqrt{F}} = \sqrt{F}$
(iv) $\sqrt{F + G} = \sqrt{\sqrt{F} + \sqrt{G}}$

Proof: (i) Assume F ⊆ G, If P is a PPOTΓI enclosing B subsequently P is a POTΓI holding F.
∴ $\sqrt{F} \subseteq \sqrt{G}$.
(ii) Let P be a PPOTΓI holding FGH ⇒ FGH ⊆ P ⇒ F ⊆ P or G ⊆ P orH ⊆ P
⇒ F⋂G⋂H ⊆ P. ∴ P is a PPOTΓI holding F⋂G⋂H.Thus rad (F ∩ G ∩ H) ⊆ rad(FGH).
Currently P be a PPOTΓI having F⋂G⋂Hi
Next F⋂G⋂H ⊆ P ⇒ FGH ⊆ F ∩ G ∩ H ⊆ P ⇒ FGH ⊆ P.
Consequently P is a PPOTΓI enclosing FGH.
Thus $rad(FGH) \subseteq rad\,(F \cap G \cap H). \Rightarrow rad(FGH) = rad\,(F \cap G \cap H)$.
As F⋂G⋂H ≠ ∅, It is obviously F⋂G⋂H is a POTΓI in T.
Let x ∈ $\sqrt{F \cap G \cap H}$. Afterwards ∃ an odd n ∈ N ∋ $w^n \in F \cap G \cap H$.
So $w^n \in F, w^n \in G, w^n \in H, \Rightarrow w \in \sqrt{F}, w \in \sqrt{G}$, & $w \in \sqrt{H}$. Thus $w \in \sqrt{F} \cap \sqrt{G} \cap \sqrt{H}$.
Consequently $w \in \sqrt{F} \cap \sqrt{G} \cap \sqrt{H}$, ∃ n, m, p ∈N ∋ $w^n \in G, w^m \in F, w^p \in H$.
Clearly $w^{n+m+p} \in F \cap G \cap H$; $n + m + p \in N \Rightarrow w \in \sqrt{F \cap G \cap H}$.
∴ if F⋂G⋂H ≠ ∅, next $\sqrt{F \cap G \cap H} = \sqrt{F} \cap \sqrt{G} \cap \sqrt{H}$.
iii) $\sqrt{F}$ = The intersection of all prime Γ- ideals of T holding F.
Currently $\sqrt{\sqrt{F}}$ = all PPOTΓI's of T whose intersection holding $\sqrt{F}$
= all PPOTΓI's of T whose intersection holding $F = \sqrt{F}$
∴ $\sqrt{\sqrt{F}} = \sqrt{F}$
iv) Via condition (i), $F + G \subseteq \sqrt{F} + \sqrt{G}$ & $\sqrt{F + G} \subseteq \sqrt{\sqrt{F} + \sqrt{G}}$
Also by condition (i) we have $\sqrt{F} + \sqrt{G} \subseteq F + G$
& thus by condition (ii), $\sqrt{\sqrt{F} + \sqrt{G}} \subseteq \sqrt{\sqrt{F + G}} = \sqrt{F + G}$

Therefore $\sqrt{F+G}=\sqrt{\sqrt{F}+\sqrt{G}}$.

Corollary 2.16: If L, M & N are any 3 POTΓI's of a POTΓS T, then thes conditions are equivalent

(i) $L \subseteq M \Rightarrow \sqrt{(L]} \subseteq \sqrt{(M]}$

(ii) If $L \cap M \cap N \neq \emptyset$ next $\sqrt{(L](M](N]} = \sqrt{(L] \cap (M] \cap (N]} = \sqrt{(L]} \cap \sqrt{(M]} \cap \sqrt{(N]}$

(iii) $\sqrt{\sqrt{(L]}} = \sqrt{(L]}$

(iv) $\sqrt{(L]+(M]} = \sqrt{\sqrt{(L]}+\sqrt{(M]}}$

Theorem 2.17: If C is a POTΓI of a POTΓS T next $\sqrt{(C]}$ is semiprime.

Proof: Via Th.1.23, $\sqrt{(\mathbf{C}]}$is semiprime**.**

Theorem2.18: A POTΓI V of POTΓS T is a SPPOTΓI iff $\sqrt{V}=V$.

Proof: Presume V is a SPPOTΓI. Obviously $\sqrt{V} \supseteq$ V

Presume if possible $\sqrt{V} \nsubseteq V$. Allow $u \in \sqrt{V},\ \&\ u \notin V$.

Currently $u \notin V \Rightarrow u \in T \backslash V$ & V is semiprime , via Th. 1.25, T\V is a PO-n-System. Via Th.1.24, ∃ a PO-m- system M ∋ $u \in M \subseteq T \backslash V$, $V \subseteq T \backslash M$ & at present T\M is a PPOTΓI, $u \notin T \backslash M$. It is a disagreement. $\therefore \sqrt{V} \subseteq V$.

$\therefore V=\sqrt{V}$. On the opposing presume Q is a POTΓI of T ∋ $V=\sqrt{V}$.

Via corollary 2.17, $\sqrt{V}$ is a SPPOTΓI. $\therefore$ V is semiprime.

Corollary2.19: A POTΓI V of a POTΓS T is a semi prime PO-ternary Γ-ideal iff V is the intersection of all PPOTΓI includes V.

Proof: Via Th.2.18., V is semiprime iff V is all PPOTΓI's whose intersection of T holds V.

Corollary2.20: If D is a POTΓI of a POTΓS T, next $\sqrt{D}$ is the smallest SPPOTΓI holding D.

Proof*:* Here $\sqrt{D}$ is all PPOTΓI's holding D whose intersection in T.

As of PPOTΓI's whose intersection is semiprime, $\sqrt{D}$ is semiprime.

Auxiliary, let V be any SPPOTΓI holding D, i.e.$V \supseteq D \ \therefore \sqrt{V} \supseteq \sqrt{D}$

As V is semiprime, via Th.3.18, $V=\sqrt{V} \ \therefore\ V \supseteq \sqrt{D}$.

Consequently $\sqrt{D}$ is the smallest SPPOTΓI of T holding D.

Theorem 2.21: If Q is a PPOTΓI of a POTΓS T next $\sqrt{Q^n}=Q\ \forall$ odd $n \in N$.

Proof: To establish $\sqrt{(Q)^n}=Q$ by using induction on n

First we confirm $\sqrt{Q}=Q$.

As Q is a PPOTΓI $Q \subseteq \sqrt{Q} \subseteq Q$

$\Rightarrow \sqrt{Q}=Q$.

Assume that $\sqrt{Q^k}=Q$ for odd natural number k ∋ $1 \leq k < n$.

presently $\sqrt{Q^{k+2}}=\sqrt{Q^k.Q.Q}=\sqrt{Q^k} \cap \sqrt{Q} \cap \sqrt{Q}=\sqrt{Q} \cap \sqrt{Q} \cap \sqrt{Q}=\sqrt{Q}=Q$.

$\therefore \sqrt{Q^{k+2}}=Q$.

By induction $Q=\sqrt{Q^n}\ \forall$ odd $n \in N$.

Theorem 2.22: In a POTΓS T amid identity there is a unique max. POTΓI M ∋ $\sqrt{M^n}$=M ∀ odd n∈N.

Proof : As T holds identity , T is a GIPOTΓS. As M is a maximal POTΓI**,** via Th.1.24 , M is Prime.

Via Th.2.21, M=$\sqrt{M^n}\ \forall\ n \in N$.

Theorem 2.23: If C is a POTΓI of a POTΓS T afterward $\sqrt{C} = \{v \in T:$ each m − system of T holding v meets C$\}$ i. e. , $\sqrt{C} = \{v \in T: M(v) \cap C \neq \emptyset\}$

Proof: Assume $v \in \sqrt{C}$. Permit M be a PO-m-system holding v

Afterward T\M is a PPOTΓI & $v \notin T\backslash M$. If $M\epsilon \cap C = \emptyset$ followed by $C \subseteq T\backslash M$,

As T\M is a PPOTΓI holding C, $\sqrt{C} \subseteq T\backslash M$ & $v \in T\backslash M$.

It is a disagreement. $\therefore M(v) \cap C \neq \emptyset$. Thus $v \in \{v \in T: M(v) \cap C \neq \emptyset\}$.

On the contrary presume $v \in \{v \in T: M(v) \cap C \neq \emptyset\}$. If possible presume $v \notin \sqrt{C}$, $\exists$ a PPOTΓI holding C ∋ $v \notin P$. Currently T\P is a PO-m-system & $v \in T\backslash P$.

$C \subseteq P \Rightarrow T\backslash P \cap C = \emptyset \Rightarrow \{v \in T: M(v) \cap C \neq \emptyset\}$.

Obviously a disagreement.

So $v \in \sqrt{C}$. $\therefore \sqrt{C} = \{v \in T: M(v) \cap C \neq \emptyset\}$.

CONCLUSION

In this research work provided a structured examination of Γ-radicals in PO-ternary Γ-semirings, Emphasizing their fundamental properties and their influence on algebraic structure. The study detailed the interplay between Γ-radicals, Ideals, Congruences, and the underlying partial order, Identifying both similarities and differences with classical radical frameworks. Conditions governing the existence and invariance of radicals, especially under homomorphisms, were established. The results enhance the theoretical foundation of PO-ternary Γ-semirings and support further advancements in radical theory for ordered ternary algebraic systems.

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