

Deconvolution Based Measurement Of Gammaray Attenuation Coefficients With Validation Using Standard Absorbers

A.H. Deshpande¹, Sangam N. N², Ashwini U³ and C.S. Mahajan⁴

¹Deogiri Institute of Engineering and Management Studies, Chh. Sambhajinagar.

²Department of Applied Science, Government Polytechnic, Jalna, Maharashtra 431203,

³Manipal Centre for Natural Sciences (MCNS), Manipal Academy of Higher Education (MAHE), Manipal, Karnataka 576104.

⁴Department of Physics, R.G. Bagdia Arts, S. B. Lakhotia Commerce and R. Bezonji Science College, Jalna, Maharashtra 431203, India.

E-mail : archanadeshpande@dietms.org

Abstract: During the measurement of attenuation parameters, the recorded gamma ray spectra are often significantly influenced by multiple factors, including the presence of multi-gamma emitting radio-nuclides, broadening of photo-peaks due to poor detector resolution and additional contributions from Compton scattered photons originating from the collimator walls. To overcome these limitations, the present work employs the Maximum Likelihood Expectation Maximization (MLEM) deconvolution technique to unfold experimentally measured NaI (Tl) gamma ray spectra in the energy range of 123–1330 keV. The detector response is modelled under the assumption that the scintillator and photomultiplier coupling controls the Gaussian broadening of the photopeak. Response-based deconvolution is used to retrieve the genuine photo-peak counts from Monte Carlo-generated spectra, which by their very nature have poor resolution. By comparing the experimentally determined mass attenuation coefficients (μ/ρ) for aluminium absorbers with the conventional theoretical values given by Hubbell and Seltzer¹ over the energy range of 123–1330 keV, the efficacy of this method is confirmed, showing good agreement.

Keywords: MLEM deconvolution, mass attenuation coefficient, gamma ray, and NaI (Tl) detector

1. Introduction:

Numerous disciplines, including nuclear physics, radiation safety, medical diagnostics, industrial inspection, and material characterization, depend heavily on gamma radiation. For quantitative analysis in these applications, precise measurement of photon interaction characteristics is crucial, especially the mass attenuation coefficient.

The precise identification of net photo-peak areas in the recorded gamma ray spectra is crucial to the accuracy of such measurements. However, a number of factors, such as electronic noise, pile-up effects, Compton-scattered photons, and low detector resolution, frequently alter the recorded spectra in actual measurements. In multi-gamma emitter nuclides, where overlapping photo-peaks and Compton continua significantly increase peak area estimation uncertainty, these effects become more pronounced. Consequently, empirically measured attenuation coefficients may differ from theoretical values by a few percent, especially at the lower energy side. In order to overcome these constraints, a number of deconvolution methods have been put forth and put into practice, allowing for the measurement of mass attenuation coefficient values that deviate from theoretical values^{1, 7, 10} by 2% to 3%

P. Paatero (1974)² conducted early research on a number of deconvolution techniques using iterative harmonic analysis techniques and generalized least-squares formulations. While later studies investigated different statistical and iterative deconvolution algorithms to increase peak separation and counting accuracy, Kennett et al. (1978)³ used

Bayesian approaches for spectrum unfolding. Later studies that confirmed maximum-likelihood approaches as reliable instruments in gamma-ray spectroscopy were made possible by L Bouches (1995)⁵, which investigated the efficacy of several deconvolution schemes in high signal-to-noise circumstances. Braian L. Evan (1999)⁶ maximum likelihood deconvolution approach to enhance photopeak resolution in Compton-scattered gamma-ray spectra. M. Sohelur Rahman et al. (2009)¹⁰ demonstrated that applying Maximum likelihood expectation-maximization (MLEM) deconvolution to NaI(Tl) detector spectra significantly enhances photo-peak resolution by effectively suppressing the Compton backgrounds, resulting in a substantial improvement in peak-to-total ratios for gamma ray measurements in water monitoring applications. A three-sigma deviation criterion, accounting for the spectral shape and energy range, is typically employed in the Maximum Entropy method when the signal-to-noise ratio is high. Zeyu Wang et al. (2019)¹³ developed a hybrid spectrometric analysis approach Richardson Lucy deconvolution combined with fuzzy inference techniques to accurately identify radionuclides from overlapping gamma ray spectra acquired using poor resolution detectors, demonstrating its effectiveness for radionuclide discrimination in complex marine environments. L J Meng (2000)⁷ evaluate complex spectra with MLEM, MEM and Linear Regularization method with considering performance parameters to allow a direct assessment of the quality of the deconvolved spectra. More recently, Monte Carlo-assisted response function modelling combined with deconvolution has been shown to significantly enhance spectral resolution and quantitative accuracy. Sukara and Rimjeam (2017)¹¹ employed GEANT4 based Monte Carlo simulations to investigate gamma ray attenuation of shielding materials such as lead, aluminum, and iron and validated their results through NaI(Tl) detector measurements. Sangam N. N (2019)¹² deconvolute complex spectra with MLEM for accurate measurement of mass attenuation coefficient of aluminum. M. J. Kumwenda et al. (2020, 2024)^{14,15} demonstrated that deconvolution of NaI(Tl) gamma ray spectra using direct matrix inversion and iterative unfolding methods significantly enhances photo-peak resolution, with the direct matrix inversion approach to providing superior peak recovery compared to conventional unfolding techniques. In the present work, the MLEM deconvolution technique is employed to extract accurate photo-peak intensities of gamma-ray spectra measured by NaI(Tl) detector over the energy range 123–1330 keV. The detector response function is modelled using Monte Carlo simulations, and the resulting deconvolved spectra are used to calculate mass attenuation coefficient values for aluminium absorbers. The results obtained are compared with standard theoretical values reported by Hubbell and Seltzer, demonstrating the reliability of the method. This study highlights the effectiveness of deconvolution based approaches for improving spectral quality and quantitative accuracy in attenuation measurements, particularly in systems with fair energy resolution.

2. Deconvolution Methodology

The experimentally measured spectrum in gamma ray spectrometry is a convolution of the true photon energy distribution that takes into account the detector system's intrinsic response characteristics. These characteristics include effects from electronic noise, finite energy resolution, incomplete charge collection, and internal Compton scattering in the detector and surrounding materials. As a result, deconvolution is necessary to retrieve the underlying counting statistics because the measured spectrum does not accurately depict the photon energy distribution. The convolution of the genuine photon energy distribution with the detector response function is represented by the measured histogram $M(E)$.

The response function $R(E, E_0)$ denotes the probability that a photon of true energy E_0 is recorded by the detection system with a measured energy E . Mathematically, $R(E, E_0)$

characterizes the energy dispersion of the detector and is typically modeled as a Gaussian function centered at E_0 , whose width is depend on detector energy resolution. The observed spectrum $M(E)$ is therefore expressed as the convolution of the true photon spectrum $S(E_0)$ with the detector response function, accounting for the probabilistic redistribution of photon energies during detection and represented as follows:

$$M(E) = \int_0^{\infty} R(E, E_0) \cdot S(E_0) dE_0. \quad (1)$$

Equation (1) for the measured spectrum $M(E)$ may be conveniently represented in matrix form.

$$M=RS$$

$$\begin{bmatrix} M_1 \\ \vdots \\ M_2 \end{bmatrix} = \begin{bmatrix} R_{11} & \dots & R_{1j} \\ \vdots & \vdots & \vdots \\ R_{i1} & \dots & R_{ij} \end{bmatrix} \begin{bmatrix} S_1 \\ \vdots \\ S_j \end{bmatrix} \quad (2)$$

$$M_i(E) = \sum_{j=1}^n R_{ij} S_j \quad (3)$$

Here, the quantity M_i refers to the counts recorded by the detector, while R_{ij} characterizes the probability that photons from the j^{th} energy bin are detected in the i^{th} channel, and S_j denotes the discretized incident spectrum.

The response matrix R_{ij} was constructed using Monte Carlo simulated detector response functions, incorporating energy dependent Gaussian broadening to model scintillation light statistics and noise generated by signal processing circuits. To maintain photon conservation, each matrix row was normalized.

Equation (3) is multiplied by the inverse of matrix $(R_{ij})^{-1}$ to give the discretized incident function solution.

$$S = R^{-1}M \quad (4)$$

Because of its intrinsic numerical instability, direct matrix inversion is solely provided for completeness's sake and is not used in this research. Rather, the Maximum Likelihood Expectation Maximization (MLEM) technique is used to solve the spectral inverse problem, allowing for stable reconstruction of the underlying photon spectrum. The MLEM technique iteratively improves the spectrum estimate and lessens the impact of Compton-scattered contributions from the collimator and surrounding materials by explicitly taking Poisson counting statistics into account. Because the detector response matrix is frequently Poorly conditioned and almost unique, making direct inversion extremely sensitive to statistical noise, this method is especially well-suited for gamma-ray spectroscopy. As a result, MLEM-based reconstruction improves the accuracy of quantitative attenuation measurements and produces physically consistent photopeak intensities.

Lucy (1974) developed the Maximum Likelihood Expectation Maximization (MLEM) technique to solve inverse issues involving ill-conditioned response matrices. This method iteratively maximizes the probability function under Poisson counting statistics to provide a statistically sound framework for reconstructing real spectral distributions from recorded data. The technique produces physically meaningful, non-negative answers and efficiently adjusts for stochastic variations in photon counts by working with discrete energy bins.

An iterative maximum-likelihood technique that takes into consideration the detector response and models detector counts as independent Poisson variables was used to estimate the true photon spectrum M_i and the iterative algorithm⁴. The process uses the paradigm presented by Shepp et al. (1982)⁴ to refine the spectral estimate at each iteration until convergence.

$$X_k^{(n+1)} = x_k^n \left\{ \frac{1}{\sum_{i=1}^N R_{ik}} \left[\sum_{i=1}^N R_{ik} \left(\frac{M_i}{\sum_{j=1}^M R_{ij} x_j^{(n)}} \right) \right] \right\} \quad (5)$$

The row and column positions in the answer matrix are shown here by the letters i and j. The MLEM algorithm updates the estimate $X_i^{(n+1)}$ at each iteration and is repeated until the change between consecutive iterations falls below a predefined tolerance based on the mean-square error.

$$\text{Tol} = \frac{\sum_{j=1}^N (x_{j+1}^{(n)} - x_j^{(n)})^2}{\sum_{j=1}^N (x_j^{(n)})^2} \quad (6)$$

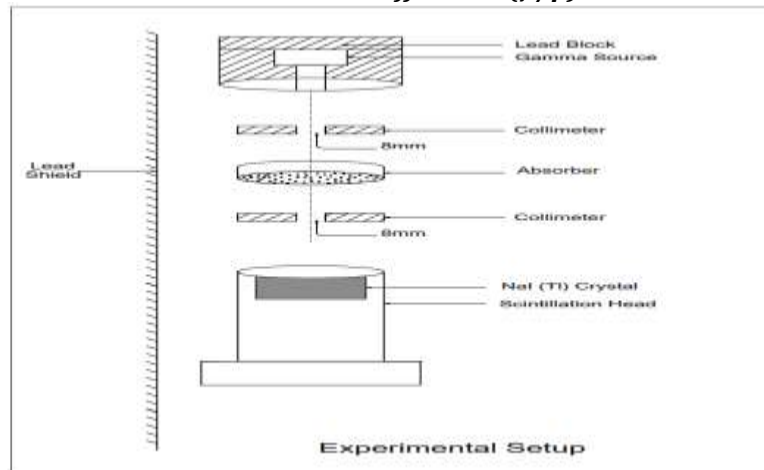
3. Experimental measurements

The experiment was carried out to evaluate the effectiveness of deconvolution techniques in improving the accuracy by performing gamma ray attenuation measurements using a NaI(Tl) scintillation detector. A well collimated narrow beam geometry (Fig:1) was employed to minimize scattering contributions and to ensure reproducible measurement conditions. A scintillation detector of having plane NaI(Tl) crystal of dimensions 1.75" × 2" was coupled to a 4k-multichannel analyzer (MCA) for spectral data acquisition. The detector was placed 24 cm from the source using an 8 mm collimator assembly. Standard radio-active sources emitting gamma rays in the energy range 123–1330 keV were used. The spectra were accumulated for 600 seconds to optimized photon counts with eliminating dead time effects.

The photo-peak of interested gamma ray energies were recorded both with and without aluminium absorber placed between the source and detector. The recorded gamma ray spectra exhibit significant broadening of photo-peaks due to the contribution of Compton scattered photons, which results in peak distortion and energy shifting. To address this effect, the net photo-peak area, the FWHM and energy resolution were determined after MLEM deconvolution over 123–1330 keV. The deconvolution process effectively reconstructs the true photo-peak by suppressing the Compton scattering from the wall of collimator. The experimental response of the NaI(Tl) detector was therefore established by calibrated gamma ray sources, forming the basis for subsequent deconvolution analysis.

To produce realistic broadened spectra, the GEB parameters were integrated into the simulation after being calculated from experimentally recorded FWHM values (Table 1). The deconvolution analysis then employed these simulated spectra as input.

Fig. 1. The experimental geometry set up to measure the aluminum absorber's mass attenuation coefficients (μ/ρ).



The iterative MLEM computations were carried out using MATLAB. As the system matrix, a simulated detector response was used to deconvolve the experimentally obtained spectra. By optimizing the likelihood function under Poisson counting statistics, the method iteratively recreated the actual photon energy distribution. Once steady convergence was attained, the final deconvoluted spectra were obtained. Convergence was tracked by analyzing variations between iterations.

After the extraction of precise photo-peak regions from the reconstructed spectra, mass attenuation coefficients for aluminum were calculated using FWHM and peak centroid parameters. The experimentally measured values were compared with theoretical data from standard NIST (1995) databases to validate the reliability of the deconvolution approach.

This combined experimental computational framework effectively corrects for detector induced spectral distortions and enhances the accuracy of gamma ray attenuation measurements, demonstrating the suitability of the MLEM based approach for high precision spectroscopic studies. The energy response function of detector is given by

$$f(E) = C \cdot e^{-\left(\frac{E-E_0}{A}\right)^2} \quad (7)$$

where E represents broadened energy, E_0 be the original tally energy, C used for normalization factor and A characterizes Gaussian peak width. A is given as,

$$A = \frac{\text{FWHM}}{2\sqrt{\ln 2}} \quad (8)$$

The GEB parameters were determined by using equation (9) fitting of FWHM values of various gamma sources carried out.

$$\text{FWHM} = \sqrt{a^2 + b^2 E + c^2 E^2} \quad (9)$$

The experimentally obtained FWHM values were plotted as a function of energy and a nonlinear least square fitting procedure was applied using OriginPro software to determine the best-fit values of the parameters a, b, and c by using equation (9). The nonlinear fitting procedure yielded finite, positive values for all fitting parameters (a, b and c), confirming the physical validity of the model, as shown in Figure 1. The Gaussian Energy Broadening (GEB) option was then used to apply the retrieved GEB parameters in the Monte Carlo simulation (MCNPX). As a result, the detector response, including peak broadening and energy dispersion effects seen in experimental data, could be accurately replicated in the simulated spectra. By using these parameters, the simulated and experimentally recorded spectra were consistent, which allowed for precise photo-peak shape modelling and trustworthy attenuation coefficient estimation. Plot of graph 2 shows comparison between convoluted and deconvoluted spectra for doublets peaks at 1170 and 1330 keV for gamma energy source Co^{60} .

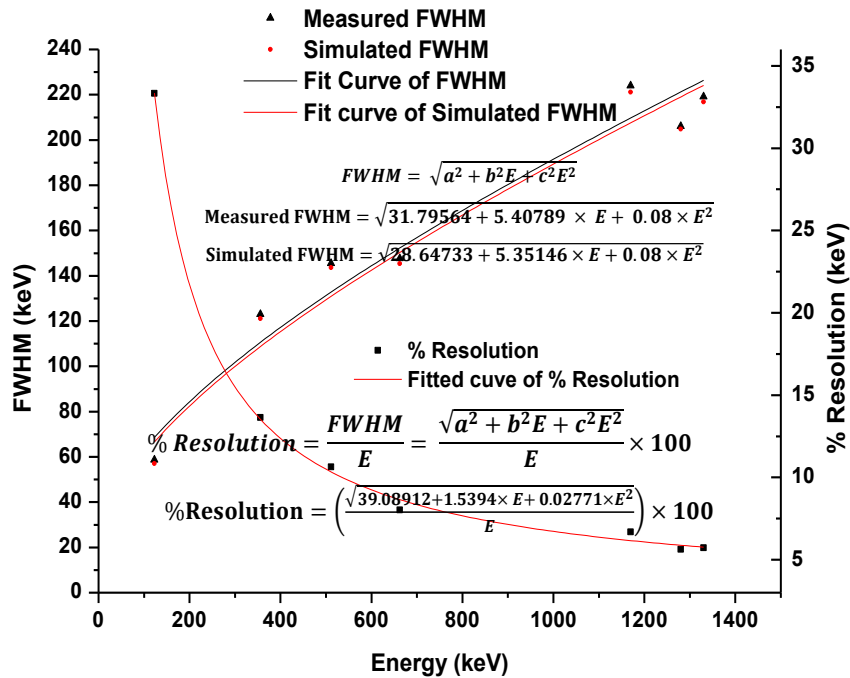
In the present work, mass attenuation coefficients of pure AR-grade aluminum foils were determined using a narrow-beam geometry identical to that described by Sangam (2019), ensuring optimal experimental conditions (Fig. 1). In a transmission experiment, the number of gamma photons detected after the absorber decreases exponentially with increasing absorber thickness, consistent with the Beer–Lambert attenuation principle.

$$I = I_0 e^{-\mu x} \quad (10)$$

Table 1. :FWHM and resolution of gamma ray photo-peak using measured MCNPX simulated and deconvoluted algorithm.

Nuclide/E nergy (keV)	Peak Channel No. (E _{DECON})	Measured Spectrum FWHM (keV)	Deconvoluted Spectrum FWHM (keV)	Resolutio n(%) (keV)	Photo- peak Energy Shift (E _{DECON} - E _{MCNpx}) (keV)
Co ⁵⁷ (123)	171	58.665	56.986	33.325	11
Ba ³³ (356)	887	122.953	121.023	13.641	24
Na ²² (511)	1349	145.531	143.564	10.642	65
Cs ¹³⁷ (662)	1810	147.602	145.367	8.031	12
Co ⁶⁰ (1170)	3303	224.004	221.065	6.694	69
Na ²² (1280)	3631	206.043	204.754	5.639	34
Co ⁶⁰ (1330)	3792	219.056	216.872	5.719	22

Graph 1: Measured FWHM and resolution of gamma ray photo-peaks for gamma energy range



123-1330 keV.

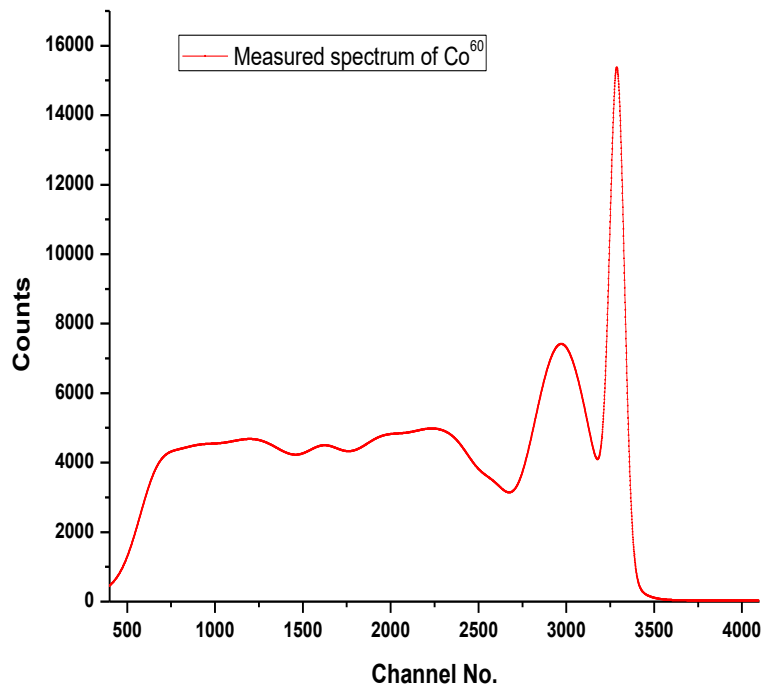
The measured experimental mass attenuation coefficient values and reported values¹ after deconvolution of spectra are shown in Table 2. The resolution of NaI(Tl) detector depends upon the scintillation electrical properties and photo-multiplier tube (PMT) assembly.

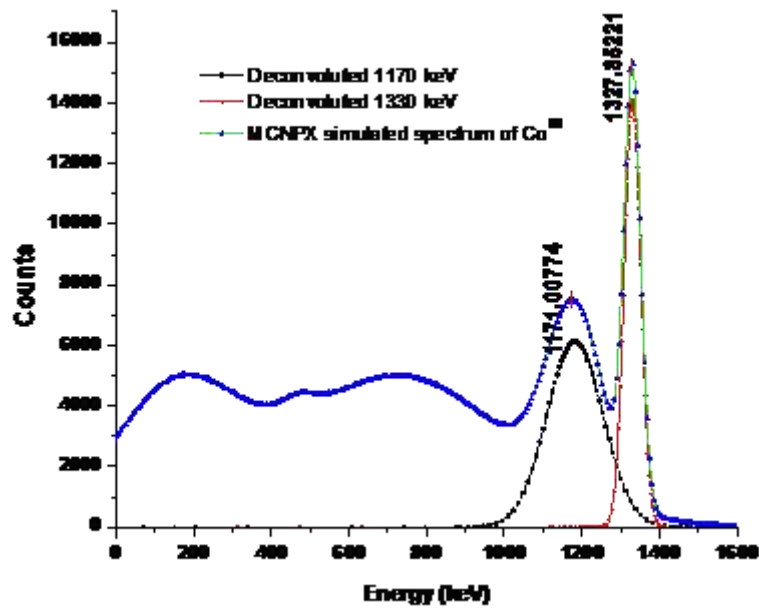
Table 2. : Measured experimental and theoretical and values of mass attenuation coefficients (μ/ρ) for standard aluminum absorber for MCNPM and Deconvoluted simulation.

Gamma Energy (keV)	Mass attenuation coefficients(μ/ρ) (cm ² /gram)			Error ^{1*} (%)
	Experimental values ¹ (MCNPX)	Experimental values ² (DECON)	Theoretical value ¹	
123	0.15195 -----	0.15630	0.15340 ¹	-1.8905
356	0.09583 0.09578 ³	0.09603	0.09730 ¹	1.3052
511	0.08610 0.08466 ²	0.08478	0.08367 ¹	-1.3266
662	0.07553 0.07547 ²	0.07537	0.07466 ¹	-0.9510
1170	0.05770 0.05620 ²	0.05690	0.05683 ¹	-0.1232
1280	0.05387 0.05102 ²	0.05309	0.05541 ¹	4.1870
1330	0.05219 0.05298 ²	0.05280	0.05325 ¹	0.8451

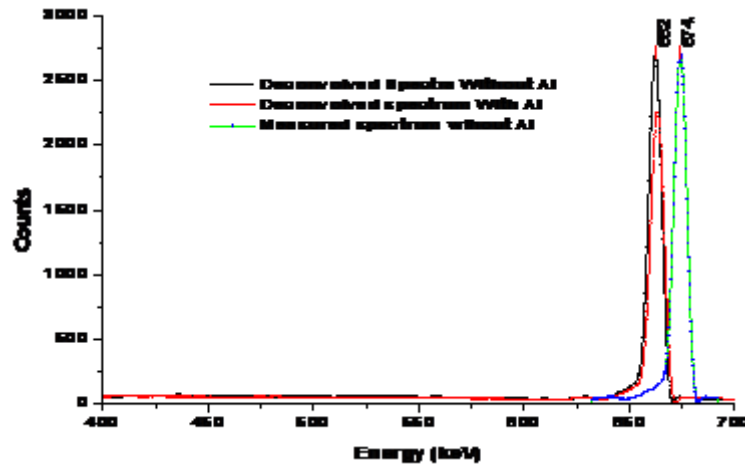
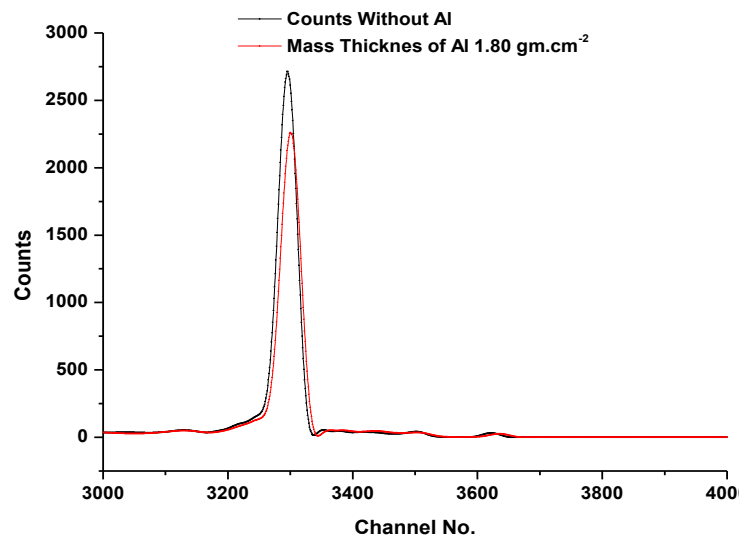
(*Expt. values of this work, ¹Hubble and Seltzar (1995), ²M.T. Teli (2001))

Graph 2 : A experimental simulated MCNPX and deconvoluted PHA spectrum for ⁶⁰Co gamma rays source .

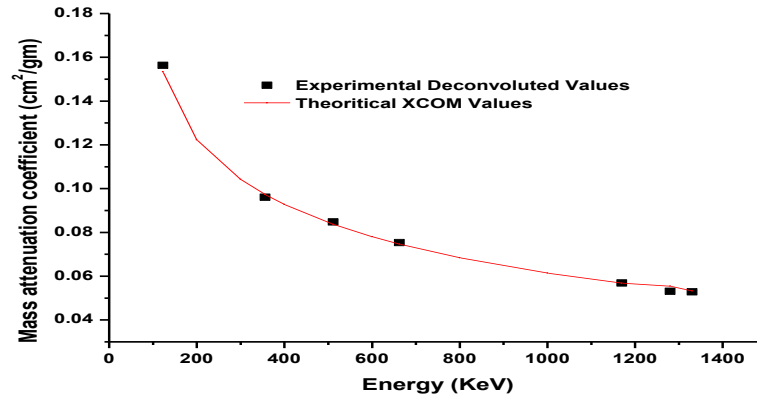




Graph 3 : A experimental simulated MCNPX and deconvoluted PHA spectrum for ^{137}Cs gamma rays source .



Graph 4: Plot of measured mass attenuation coefficients (μ/ρ) values for aluminum after PHA deconvolution with theoretical XCOM values at various gamma ray energies range



4. Results and discussion

The relevance to perform attenuation experiment over the gamma ray energy 123-1330 keV is to verify utility of MLEM deconvolution. Results of this experiment has presented in Table-2 compared and shows demonstrates a strong correlation with theoretical Hubbell NIST values¹. The mass attenuation coefficient values particularly at energies 123 and 1280 keV is slightly large is due to excess suppression of background counts to ensure effectiveness of MLEM data processing.

The observed FWHM and hence resolution of photo-peak before and after deconvolution are mention in Table-1 shows significantly improvement even after high signal to noise ratio and background counts. Experimental simulated MCNPX and deconvoluted photo-peaks for Cs¹³⁷ and Co⁶⁰ gamma rays are shown in Graph 1 and 2 shows considerable shifting of energy channels typically at energy 511 and 1170 keV are considerably high, because of PMT ground counts, Compton edge distribution and response function at higher energies. The application of MLEM deconvolution markedly enhances photopeak resolution, especially for multi-energy gamma emitters, thereby enabling more reliable and precise determination of attenuation parameters.

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