Analysis of Loss Mechanisms in Hemispherical Shell of Vibratory Gyroscopes

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Abstract—CVG (Coriolis Vibratory Gyroscope) is one kind of sensor in inertial sensors technology for navigation applications in aerospace field. Different factors effecting Q-factor of hemispherical shell are analyzed. Different loss mechanisms are studied and ways to reduce them are presented in this paper. The precessional phenomenon and vibration characteristics of hemispherical resonator shell are investigated in this paper. Thin shell theory and virtual displacement principle are employed to model hemispherical shell. The influence of shell's geometry parameters on its natural frequency and precessional rate is analyzed. ANSYS-finite element analyzer, is used to analyze the influence of shell's geometry, physical parameters on optimal Q-factor.

Keywords—Hemispherical Shell; flexural vibration; q-factor; resonant frequency.

I. INTRODUCTION

Navigation is a process of guiding safely and efficiently from one destination to another destination. Navigation in detail expands the process of "Where am I" and "How do I get to my destination" with "reference to What". Navigation can be defined broadly as the determination of physical body's position and velocity and guiding along the defined path relative to reference co-ordinate system so that destination is achieved. The process of navigation requires a) determination of present position and velocity of the physical body or vehicle in motion relative to known reference system, b) modifying the path or course of the physical body or vehicle in motion, c) stabilizing the vehicle or physical body so the it does not deviate from actual path, thus Navigation includes Guidance and Control and requires sensors to determine the present position and velocity of physical body or vehicle. Conventionally there are few types of Navigation techniques used traditionally which includes Celestial navigation, Inertial navigation, Radio Navigation, Satellite navigation and Integrated Navigation. Inertial sensors such as Gyroscopes and accelerators are applied in the Inertial Navigation sphere where it is necessary to define the current location and speed of the physical object or vehicle.

Gyroscope is a sensor which measures the angle of rotation. Gyroscopes are operated in a wide range of dynamic applications that demand navigation, guidance, stabilization, orientation capabilities in transportation, space explorations, military systems, mining operations and consumer electronics. Despite significant advancements in the technological level of gyroscope industry, persistent demands for enhanced sensitivity, low cost, miniaturization, GPS/GNSS independent navigation requirements makes way for continuous novel research and development efforts in this sensor technology field.

Hemispherical Resonator Gyro (HRG) is the only Coriolis Vibratory Gyros (CVG) that has attained the performance of an inertial grade system. One of the sensing properties of gyroscopes is the standing vibration procession, and HRG is a solid-state gyroscope based on this property. It has the properties of high accuracy, high life, inherent high reliability, natural hardness of the radiation and small size.

A solid hemispherical shell shows high resistance to extension and compression, maintaining a constant total circumferential length at its rim during resonance vibrations. The thin shell walls undergo periodic deformations into specific shapes defined by standing wave patterns and associated vibrational modes. Due to mechanical stress and external force in hemispherical shell the wave pattern with two maxima and two minima at rim is formed. This wave pattern consists of two nodes and two antinodes. CVG sensor senses this standing wave pattern precision due to wave inertia property of hemispherical shell upon external rotation in this vibration pattern. When the hemispherical shell in case is rotated externally about the stem or rotation axis and the standing wave pattern generated at rim due to mechanical stress and inertia nodes and antinode's location on rim of hemispherical shell precess w.r.t. fixed location on the hemispherical shell by a factor 0.3 or depending upon the shape of resonating element. The angular rotation about the rotation axis of the CVG sensor is evaluated by detecting the change in the locations of nodes and antinodes of vibration pattern at rim in hemispherical shell.

HRG is one type of vibrating gyros that can work as an angle sensor as well as an angular rate sensor depending on its mode of operation. These two operation modes are Whole Angle (WA) mode and Force to rebalance mode (FTR). Under WA mode, the standing wave is left to process freely relative to external rotation forces of Coriolis. In FTR mode the standing wave is fixed at a specific location by applying the rebalance force. The angular rate can be estimated by the force applied to fix standing wave. In general, WA mode has relatively larger dynamic range with relatively low accuracy while FTR mode has shorter dynamics range with high measurement accuracy [3].

The shell model discussed in this paper relies on the Kirchhoff-Love hypothesis that states that lines that were originally normal to the mid-surface will be straight and normal to the mid surface after deformation, and the thickness of the shell will remain constant throughout the deformation process and the Finite Element Analysis (FEM) will be applied to investigate the effects of the influence of Physical properties of material, Geometrical parameters of shell.

II. MODEL OF RESONATOR

A. Co-ordinates and Equations of Vibration Mode:

An example of a thin axis-symmetrical hemispherical shell is illustrated in fig 1. *R* is the average radius of the shell, φ and θ are the longitudinal and circumferential coordinates, respectively. The function $\alpha(\varphi)$ defines the shell thickness at a specific angle φ. The angles $\varphi 0$ and φf correspond to the bottom and top boundary angles, respectively, which for a perfectly axisymmetric hemispherical shell range from 0° to 90°

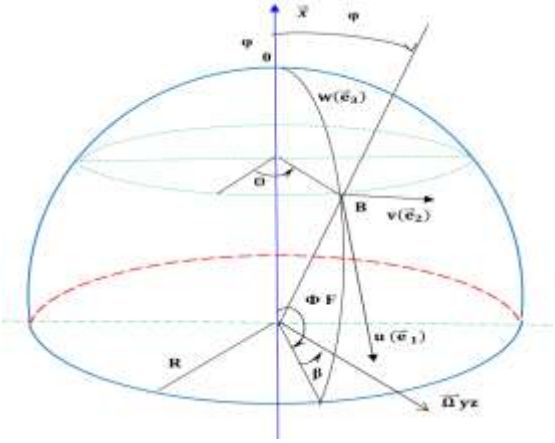


Fig.1 Illustration of shell's geometry parameters

The motion of any arbitrary point on the hemispherical shell can be expressed using a displacement field written in terms of three orthogonal directions:

$$\vec{v} = u\vec{a} + v\vec{b} + w\vec{c} \quad (1)$$

where u, v, and w represent the displacement components along the meridional (generatrix), circumferential (ring), and radial directions, respectively [8].

The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ denote the corresponding unit basis vectors associated with these directions.

When the resonator undergoes rotation with an angular velocity vector $\vec{\Omega}$, it may be decomposed into components such that:

$$\vec{\Omega} = \vec{\Omega}_x + \vec{\Omega}_{yz}$$

Under the assumption that the rotational speed is much smaller than the natural vibration frequency, i.e.,

$$\Omega^2 \ll \omega^2, \frac{\partial \Omega}{\partial t} \ll \omega^2$$

the influence of initial stress effects and angular acceleration terms can be considered negligible. Consequently, these factors do not significantly alter the modal vibration behavior of the hemispherical shell.

For an isotropic rotating hemispherical resonator, the vibration response can therefore be represented using an analytical standing-wave formulation as follows:

The vibrational displacement field of an isotropic rotating hemispherical resonator can be represented using modal standing-wave expressions. The displacement components along the meridional, circumferential, and radial directions are written as:

$$u(\phi,\theta,t) = u(\phi)\cos\, n(\theta+\psi)\cos\, \omega t v(\phi,\theta,t) = v(\phi)\sin\, n(\theta+\psi)\cos\, \omega t w(\phi,\theta,t) = w(\phi)\cos\, n(\theta+\psi)\cos\, \omega t \quad (2)$$

Here, ψ denotes the angular precession of the standing wave pattern. It evolves with time according to:

$$\psi = \int_0^t P dt, P = K\Omega_x$$

where K is referred to as the precession coefficient. The integer n indicates the number of circumferential wave lobes, while ω represents the natural resonance frequency of the shell.

The kinetic energy associated with the vibrational motion of the hemispherical shell may be expressed as:

$$K = \frac{1}{2} r^2 \rho h \int_0^{\pi/2} \int_0^{2\pi} (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\sin\, \phi d\theta d\phi \quad (3)$$

To evaluate the elastic behavior of the resonator, the strain energy stored during deformation is obtained using classical thin-shell elasticity theory based on the Kirchhoff–Love assumptions. The total strain energy is given by:

$$U = \frac{1}{2}\int_V (\sigma_\phi e_\phi + \sigma_\theta e_\theta + \sigma_{\phi\theta}\gamma_{\phi\theta}) dV \quad (4)$$

For a homogeneous isotropic material, the stress–strain relationships can be written as:

$$\sigma_\phi = \frac{E}{1-\mu^2}(e_\phi + \mu e_\theta) \sigma_\theta = \frac{E}{1-\mu^2}(e_\theta + \mu e_\phi) \sigma_{\phi\theta} = \frac{E}{2(1+\mu)}\gamma_{\phi\theta} \quad (5)$$

where E is Young's modulus and μ is Poisson's ratio.

The strain components at any point through the shell thickness may be decomposed into mid-surface contributions and bending-related curvature terms as:

$$e_\phi = \varepsilon_\phi + z\kappa_\phi e_\theta = \varepsilon_\theta + z\kappa_\theta \gamma_{\phi\theta} = \varepsilon_{\phi\theta} + 2z\tau \quad (6)$$

In these expressions, $\varepsilon_\phi$and $\varepsilon_{\phi\theta}$correspond to the membrane (in-plane) normal and shear strains on the middle surface, whereas $\kappa_\phi$and $\kappa_\theta$describe the changes in curvature. The parameter z represents the distance from the mid-surface along the shell thickness direction.

τ is the image of a middle surface twist and z are the distance of the arbitrary point on the middle surface to the respective point o the middle surface along 0and changes across the thickness. In this case when considering a hemispherical shell, the change in middle surface strain and curvature is. [7]:

$$\varepsilon_\phi = \frac{1}{r}(w + \partial\, u/\partial\, \phi), \varepsilon_\theta = \frac{1}{r}\left(\frac{1}{sin\phi}\partial\, v/\partial\, \theta\right) + u.c.tan\phi + w, \varepsilon_{\phi\theta} = \frac{1}{r}\left(\frac{1}{sin\phi}\partial\, u/\partial\, \theta + \partial\, v/\partial\, \phi - v.c.tan\phi\right), \kappa_\phi = \frac{1}{r^2}(\partial\, u/\partial\, \phi - \partial^2\, w/\partial\, \phi^2), \kappa_\theta = \frac{1}{r^2 sin\phi}\left(\frac{-1}{sin\phi}\partial^2\, w/\partial\, \theta^2 + \partial\, v/\partial\, \theta - cos\phi * \partial\, w/\partial\, \phi + ucos\phi\right) \tau = \frac{1}{r^2 sin\phi}\left(\frac{-\partial^2 w}{\partial\phi\partial\theta} + \frac{\partial w}{\partial\theta}ctan\phi + \partial\, u/\partial\, \theta + \frac{\partial v}{\partial\phi}sin\phi - vcos\phi\right) \quad (7)$$

As for free vibration of clamped free hemispherical shell, under the condition of paucity displacement, the Lord Rayleigh in extensional condition is satisfied, so the normal stress and shear stress will be approximately reduced into zero, which is:

$$\varepsilon_\phi = \varepsilon_\theta = \varepsilon_{\phi\theta} = 0 \quad (8)$$

Substituting Equations (7) & Equations (8) in Equations (2), Equation (2) can be expressed as:

$$u = Csin\phi tan^n\frac{\phi}{2}cosn\theta sin\omega_n t, v = Csin\phi tan^n\frac{\phi}{2}sinn\theta sin\omega_n t, w = -C(n + cos\phi)tan^n\frac{\phi}{2}cosn\theta sin\omega_n t \quad (9)$$

Substituting Equations (9) in Equations (3) & Equations (4), the expressions for Kinetic& Potential energy are given by:

$$K\frac{1}{2_n}^{2^{2^2}}\int_0^{\pi/2} \quad U\frac{\pi C^2 h^3 E}{6(1+\mu)r^2}^{2^{2^2}}\int_0^{\pi/2} sin^{-3}\phi tan^{2n}\frac{\phi}{2}_{max} \quad (10)$$

III. Analytic Solution

For elements on the shell, the energies include virtual energy δW_T due to total elastic force, Virtual energy δWe due to Coriolis force due to angular velocity $\vec{\Omega}$, virtual energy δT due to inertia force.

The virtual displacement principle states that,

$$\delta W_T + \delta W_e + \delta T = 0$$

A. Resonator Frequency

At resonance: K. E_{max} = P. E_{max}

The natural resonator frequency can be derived as:

$$\omega_n = \frac{n(n^2-1)h}{r^2}\left(\frac{E}{3(1+\mu)\rho}\frac{\int_0^{\pi/2} sin^{-3}\phi tan^{2n}\frac{\phi}{2}d\phi}{\int_0^{\pi/2}\left\{(n+cos\phi)^2+2sin^2\phi tan^{2n}\frac{\phi}{2}d\phi\right\}}\right) \quad (11)$$

In the case of vibration mode of order n=2 of the resonator, the natural frequency is given by:

$$\omega_2 = \frac{2h}{r^2}\left(\frac{3E}{(1+\mu)\rho}\frac{I}{J}\right)^{\frac{1}{2}} \quad (12)$$

In which I and J refer to the shape of hemispherical shell are determined as follows:

$$I = \int_0^{\pi/2} sin^{-3}\phi tan^{2n}\frac{\phi}{2}d\phi; J = \int_0^{\pi/2} \quad (13)$$

B. Precession Coefficient

The precession coefficient K is given by:

$$K = \frac{P}{\Omega_x} = \frac{-2G_1}{nG_2} \quad (14)$$

Where:

$$G_1 = \int_0^{\pi/2}(n - cos\phi)\, sin^2\phi tan^n\frac{\phi}{2}d\phi - sin^3\phi tan^n\frac{\phi}{2}; G_2 = \int_0^{\pi/2} 2sin^2 d\,\phi tan^n\frac{\phi}{2}d\phi - sin\phi(n + cos\phi)tan^n\frac{\phi}{2} \quad (15)$$

By analyzing and calculating above results in theory it can be concluded that:

- Resonator frequency: The resonator frequency is higher as the number of the ring waves n increases. The lower the radius of shell the higher the resonator frequency. An increase in resonator frequency as shell becomes thicker i.e. h increases [6].
- Precession Coefficient: With increasing value of n of ring waves, precession coefficient, K decreases.

C. Thermoelastic Damping

Thermos-elastic damping is primary energy dissipation mechanisms in hemispherical resonator. It mainly arises due to the stress (cycle) during resonator vibration as shown in fig4 which induces heat flow within the material of resonator shell.Zener [5] provided a significant simplification by developing an analytical approximation. By assuming the one-dimensional model forming, heat diffusion across the beam thickness and harmonic bending strains,

$$\frac{1}{Q_{TED}} = \frac{Y\alpha^2 T_0}{\rho C_p}\frac{\omega\tau_{thermal}}{1 + (\omega\tau_{thermal})^2}$$

Where,

Y represents the modulus of Young, α is the coefficient of thermal expansion, T_0 the ambient temperature, $C_p$is the specific heat capacity of the material, ρ is the density of that material, and $\tau_{thermal}$ is the thermal transport time.

Considering the thermal path length d, thermal conductivity k,

$$\tau_{thermal} = \frac{\rho C_p d^2}{\pi^2 k}$$

The second term depends on geometry and frequency of the resonator structure. It exhibits the lorentzian behaviour.

Q_{TED} reaches minimum when

$\omega\tau_{thermal}$=1, $Q_{TED} = 2$ is minium value.

Quality factor increases if $\omega\tau_{thermal}$ increases or decreases. This behavior can be understood by considering two limiting regimes; isothermal and adiabatic. In the isothermal regime ($\omega\tau_{thermal} \ll 1$) the vibrational period is much longer than the thermal-diffusion time, allowing heat to redistribute almost instantaneously; the shell remains nearly in thermal equilibrium, resulting in low energy loss. Conversely, in the adiabatic regime ($\omega\tau_{thermal} \gg 1$) the resonator deforms so rapidly that heat has insufficient time to flow, again limiting dissipation. Thus, maximum dissipation and minimum Q_{TED} occur only in the narrow transition region around $\omega\tau_{thermal} \approx 1$. Moving away from this point in either direction reduces thermoelastic damping as shown in fig4.

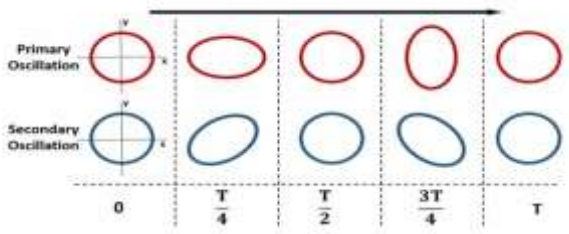


Fig.2 Illustration of vibrating shell shapes during stress cycle.

IV. FINITE ELEMENT ANALYSIS

A. Modal Analysis

Analysis of Hemispherical Shell is done using ANSYS-Finite Element Analyzer Software by consideration of geometry parameters of the resonator as in fig.3.

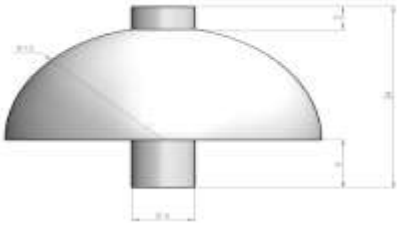


Fig.3 Illustration of shell's geometry parameters

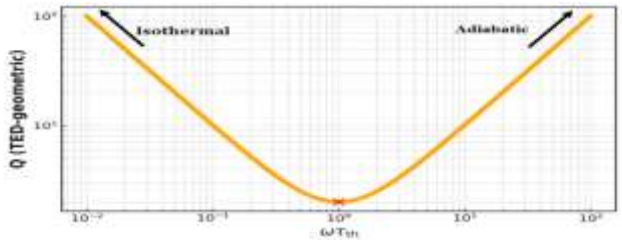
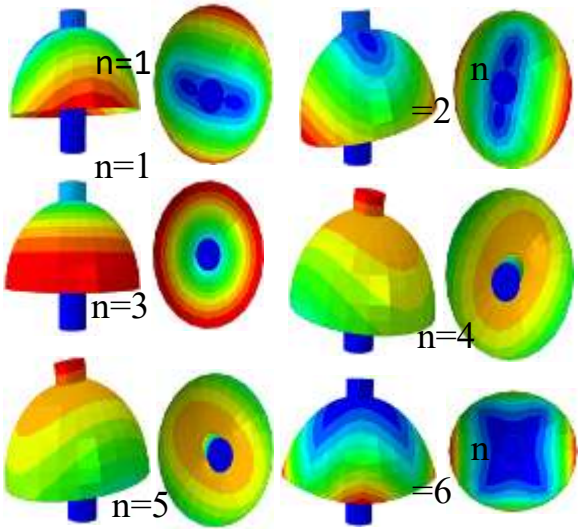


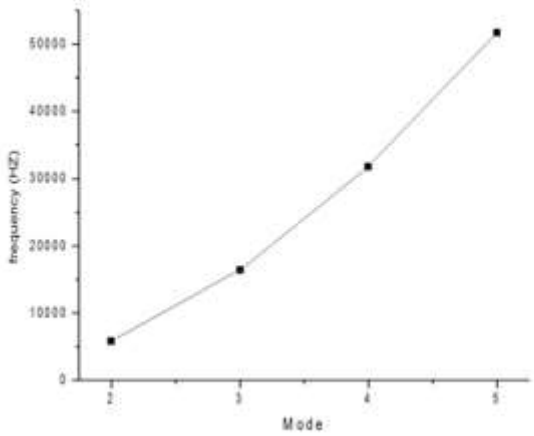
Fig.4 Illustration of Q_{TED}

TABLE I. RESONATOR SERIES OF NATURAL FREQUENCIES (HZ)

Natural frequency	Order					
	N=2	N=3	N=4	N=5	N=6	N=7
ω	6700	16567	28698	433888	60161	79497

Fig.5 Illustration of shell's modes shapes





Different forms of vibration are gained in the resonator on a series of frequencies. The higher the number of the ring waves n, the higher the frequencies will be as well, it will be difficult to excite the resonator. The more the number of the ring waves, the more complex will be the vibrating shape, the complex will be signal detection.

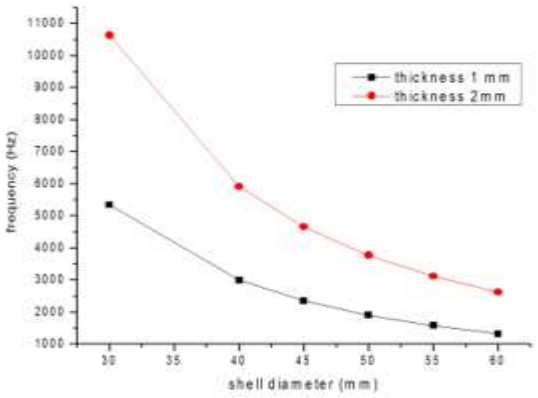
B. Geometry parameters influence on frequencies

By changing the geometry parameters, several finite element models are studied and analyzed utilizing Ansys.

TABLE II. ***SIMULATION RESULTS***

Shell Parameters	**Natural Frequency (HZ)**
φ =60mm; h=1mm	1328.6
φ =60mm; h=2mm	2624.4
φ =55mm; h=1mm	1579.6
φ =55mm; h=2mm	3121.7
φ =50mm; h=1mm	1907.4
φ =50mm; h=2mm	3776.1
φ =45mm; h=1mm	2355.8
φ =45mm; h=2mm	4665.1
φ =40mm; h=1mm	3002.3
φ =40mm; h=2mm	5921.3
φ =30mm; h=1mm	5351.9
φ =30mm; h=2mm	10637

- When the radius of shell R_1 decreases, the corresponding series of frequencies are increased.



- The shell thickness has significant impact on all the vibration modes. The n=2 mode frequencies increase as the shell becomes thicker while the distance between the neighbours is made shorter.
- In case of a reduction in stem radius R_2, all the orders of frequencies are reduced. Conversely when the stem radius is presented further each order of frequencies is amplified; however, the neighbourhood distance is reduced.

With the above result analysis, it can be inferred that the geometry parameters of the resonator play significant roles in frequency of resonator. The selection of the geometry parameters is important in order to work with the hemispherical resonator.

C. Conclusion:

The paper is about the vibrational and precessional mode of the hemispherical shell. According to the principle of virtual displacement, the kinetic equation of hemispherical shell is discussed. The parameters of the geometry of shell that influence its natural frequency and rate of precession are assumed to be an analytic solution of the model. The effect of the geometry parameters of shell to each free frequency is studied using ANSYS-Finite Element Analyzer [5].

The discussion presented in the paper could be applicable in the design of Hemispherical Shell to achieve maximum Q-factor and the study of source of error in performance analysis.

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